

$$1) \text{ الف) } |x| = t \Rightarrow y = \frac{t+1}{t-2} \Rightarrow y(t-2) = t+1 \Rightarrow yt - 2y = t+1 \Rightarrow t(y-1) = 2y+1$$

وقت!

$$\Rightarrow t = \frac{2y+1}{y-1} \quad t \geq 0 \Rightarrow \frac{2y+1}{y-1} \geq 0 \Rightarrow \frac{-\frac{1}{2}}{1} \leq \frac{1}{1} \Rightarrow R = (-\infty, -\frac{1}{2}] \cup (1, +\infty)$$

$$ب) |x| = t \quad x \neq 0 \Rightarrow y = \frac{t^2+1}{t} \Rightarrow t + \frac{1}{t} \times t \Rightarrow yt = t^2 + 1 \Rightarrow t^2 - yt + 1 = 0 \quad (1, \sqrt{2})$$

$$\Delta \geq 0 \Rightarrow (-y)^2 - 4(1)(1) \geq 0 \Rightarrow y^2 - 4 \geq 0 \Rightarrow (y-2)(y+2) \geq 0 \Rightarrow y \geq 2 \Rightarrow R = [2, +\infty)$$

$y=2 \quad y=-2$

$$2) \text{ طرفین ضرب کنیم} \Rightarrow y(x+2) = x^2 + 2x - 2 \Rightarrow x^2 + (2-y)x - (2+2y) = 0 \xrightarrow{\Delta \geq 0} (2-y)^2 - 4(-2-2y) \geq 0$$

$$\Rightarrow y^2 - 4y + 4 + 8 + 8y \geq 0 \Rightarrow y^2 + 4y + 12 \geq 0 \Rightarrow (y+2)(y+6) \geq 0 \Rightarrow \left. \begin{matrix} y \leq -6 \\ y \geq -2 \end{matrix} \right\} R = (-\infty, -6] \cup [-2, +\infty)$$

$$3) D \Rightarrow x^2 - x + 1 = 0 \xrightarrow{\Delta} (-1)^2 - 4 = -3 < 0 \Rightarrow D = R$$

$$R \Rightarrow \text{طرفین ضرب کنیم} \Rightarrow y(x^2 - x + 1) = x^2 - x + \frac{1}{y} \Rightarrow yx^2 - yx + y = x^2 - x + \frac{1}{y}$$

$$\Rightarrow (y-1)x^2 + (1-y)x + (y - \frac{1}{y}) = 0 \xrightarrow{\Delta \geq 0} (1-y)^2 - 4(y-1)(y - \frac{1}{y}) \geq 0$$

$$\Rightarrow y^2 - 2y + 1 - (4y^2 - 2y + 4) \geq 0 \Rightarrow -3y^2 + 2y - 3 \geq 0 \Rightarrow 3y(1-y) \geq 0$$

$$\Rightarrow 0 \leq y \leq 1 \Rightarrow R = [0, 1]$$

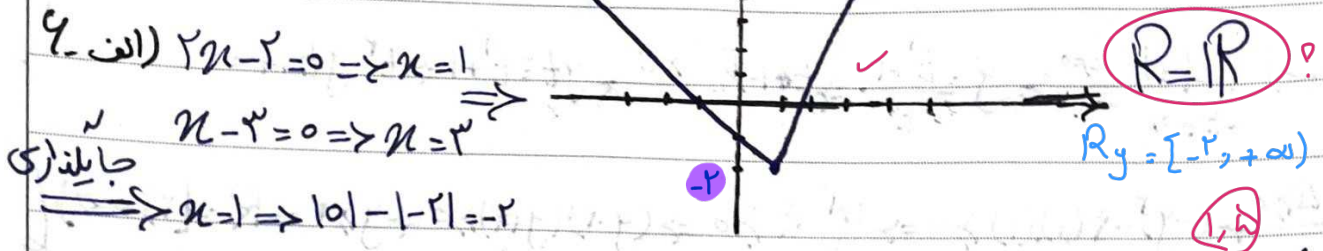
$$4) \text{ الف) } y = \sqrt{-(x-2)^2 + 3} \Rightarrow -(x-2)^2 + 3 \geq 0 \Rightarrow 0 \leq y \leq \sqrt{3} \Rightarrow R = [0, \sqrt{3}]$$

$$ب) y = \sqrt{2(x-2)^2 - 1} \geq -1 \Rightarrow -2 \leq y < +\infty \Rightarrow R = [-2, +\infty)$$

$$5) \text{ الف) } x^3 - x + 1 \xrightarrow{x^3} \left\{ \begin{matrix} x \rightarrow -\infty \Rightarrow f(x) \rightarrow -\infty \\ x \rightarrow +\infty \Rightarrow f(x) \rightarrow +\infty \end{matrix} \right\} \Rightarrow D = R$$

$$b) \frac{x^2+1}{x} = x + \frac{1}{x} \begin{cases} x > 0 \Rightarrow t = x + \frac{1}{x} \Rightarrow tx = x^2 + 1 \Rightarrow x^2 - tx + 1 = 0 \Rightarrow t^2 - 4 \geq 0 \Rightarrow t \geq 2 \vee t \leq -2 \\ x < 0 \Rightarrow t \leq -2 \end{cases}$$

$$\Rightarrow \log_2 \left(\frac{x^2+1}{x} \right) \geq \log_2 2 \Rightarrow y \geq 1 \Rightarrow R = [1, +\infty)$$

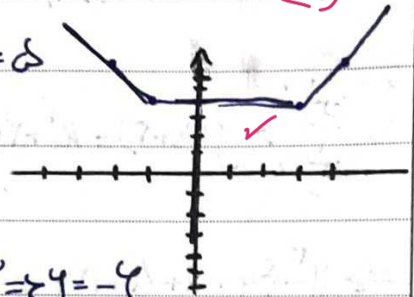


$$x = 3 \Rightarrow |4| - |0| = 4, x = 0 \Rightarrow y = -1, x = 4 \Rightarrow y = 2$$

$$b) x - 3 = 0 \Rightarrow x = 3$$

$$x + 1 = 0 \Rightarrow x = -1$$

$$R = [4, +\infty)$$



$$\Rightarrow x = 3 \Rightarrow y = 4, x = -1 \Rightarrow y = 4, x = 4 \Rightarrow y = 2, x = -2 \Rightarrow y = -4$$

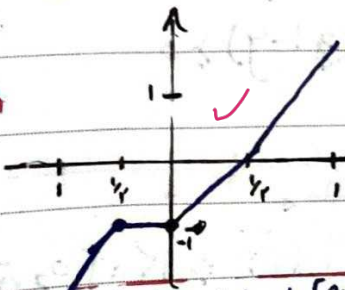
$$x = -1 \Rightarrow x < -1 \Rightarrow (2-x) - (-2x-2) \leq 1 \Rightarrow x+4 \leq 1 \Rightarrow (-\infty, -3]$$

$$-1 < x < 2 \Rightarrow (2-x) - (2x+2) \leq 1 \Rightarrow x > -\frac{1}{3} \Rightarrow [-\frac{1}{3}, 2)$$

$$x > 2 \Rightarrow (x-2) - (2x+2) \leq 1 \Rightarrow x \geq -5$$

$$\Rightarrow (-\infty, -3] \cup [-\frac{1}{3}, +\infty)$$

$$x = -\frac{1}{3} \Rightarrow \begin{cases} x < -\frac{1}{3} \Rightarrow y = 3x - (2x-1) - x = x+1 \Rightarrow x = -\frac{1}{3}, y = -\frac{2}{3} \\ -\frac{1}{3} < x < 0 \Rightarrow y = 3x - (2x+1) - x = -x-1 \\ x > 0 \Rightarrow y = 3x - (2x+1) + x = x-1 \Rightarrow x = 0, y = -1 \end{cases}$$



$$\Rightarrow R = \mathbb{R}$$

$$-2 \leq x < -1 \Rightarrow [x] = -2, 2-x \leq 0 \Rightarrow [2-x] = 0 \Rightarrow y = -2+0 = -2$$

$$-1 \leq x < 0 \Rightarrow [x] = -1, 2-x \leq 0 \Rightarrow [2-x] = 0 \Rightarrow y = -1+0 = -1$$

$$0 \leq x < 1 \Rightarrow [x] = 0, 2-x \leq 0 \Rightarrow [2-x] = 0 \Rightarrow y = 0+0 = 0$$

$$1 \leq x < 2 \Rightarrow [x] = 1, 2-x \leq 0 \Rightarrow [2-x] = 0 \Rightarrow y = 1+0 = 1$$

$$2 \leq x < 3 \Rightarrow [x] = 2, 2-x \leq 0 \Rightarrow [2-x] = 0 \Rightarrow y = 2+0 = 2$$

$$3 \leq x < 4 \Rightarrow [x] = 3, 2-x \leq 0 \Rightarrow [2-x] = 0 \Rightarrow y = 3+0 = 3$$

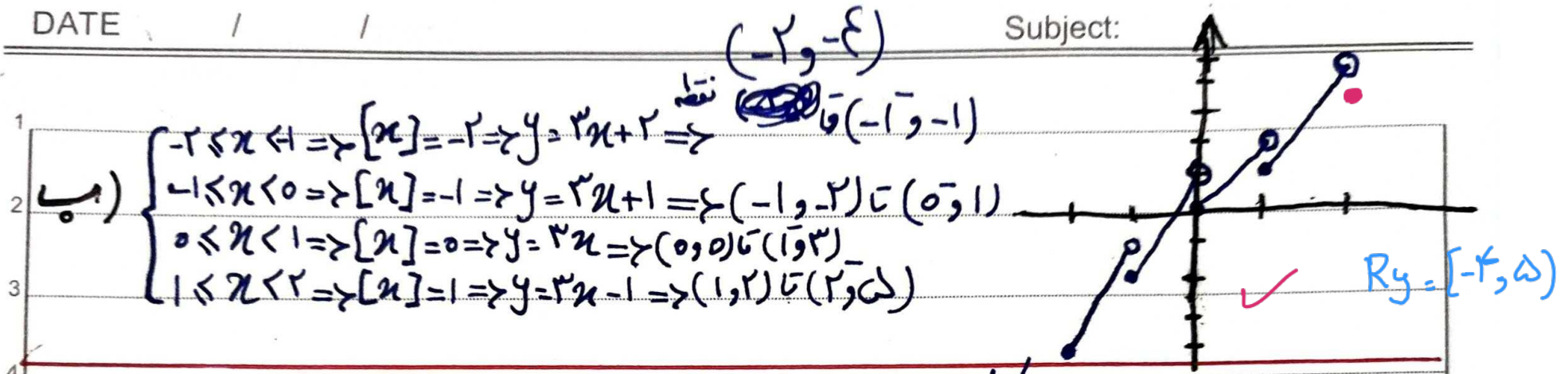
$$4 \leq x < 5 \Rightarrow [x] = 4, 2-x \leq 0 \Rightarrow [2-x] = 0 \Rightarrow y = 4+0 = 4$$

$$5 \leq x < 6 \Rightarrow [x] = 5, 2-x \leq 0 \Rightarrow [2-x] = 0 \Rightarrow y = 5+0 = 5$$

برد از خواستیم:

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$$\sin x = t \Rightarrow y = t^2 - t + 1 \quad \text{صیغه کلی} \quad y = (t - \frac{1}{2})^2 + \frac{3}{4}$$

$0 \leq t \leq 1$

مینه نسبت $y \geq \frac{3}{4}$

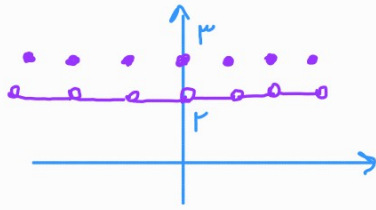
(2)

Max $t = -1 \rightarrow y = 1 + 1 + 1 = 3 \Rightarrow R = [\frac{3}{4}, 3]$

(ب) $t = \sin^2 x \Rightarrow y = t^2 + t + 1 \Rightarrow \begin{cases} t = 0 \Rightarrow y = 1 \\ t = 1 \Rightarrow y = 3 \end{cases} \Rightarrow R = [1, 3]$

$0 \leq t \leq 1$

٩الف) $y = [u] + [-u] + r$



$R_y = \{r, r\}$