

شماره تکلیف:  $\Delta$

مقطع: دوازدهم پسرانه

18,5

نام و نام خانوادگی: حسام رفیع زاده

$$D_{g \circ f} = \{x \in D_f \mid f(x) \in D_g\} \quad f(x) = \sqrt{\log_2(x-1)} \quad , \quad g(x) = \sqrt{-x^2 + 4x - 4}$$

$$D_f: \log_2(x-1) \geq 0 \quad \rightarrow \quad D_f: [2, +\infty) \quad (1)$$

$$D_g: -x^2 + 4x - 4 \geq 0 \rightarrow x^2 - 4x + 4 \leq 0 \rightarrow (x-2)^2 \leq 0 \rightarrow x=2 \quad (2)$$

$$f(x) \in D_g \xrightarrow{(1)} \sqrt{\log_2(x-1)} = 2 \xrightarrow{(2)} \log_2(x-1) = 4 \rightarrow x-1 = 17 \rightarrow x = 18$$

$$D_{g \circ f} = D_f \cap \{x \in D_f \mid f(x) \in D_g\} \Rightarrow D_{g \circ f} = \{18\} \quad (3)$$

$$f(x) = 2x - 5$$

$$g(x) = x^2 - 3x + 1$$

$$f \circ g(x) = g \circ f(x) \rightarrow 2(x^2 - 3x + 1) - 5 = (2x - 5)^2 - 3(2x - 5) + 1$$

$$\rightarrow 2x^2 - 6x + 2 - 5 = 4x^2 - 12x + 11 \rightarrow 2x^2 - 6x + 11 = 0$$

$$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 = (5)^2 - 2p = \left(\frac{-b}{a}\right)^2 - 2 \times \frac{c}{a} = (1.5)^2 - 2 \times \frac{11}{2} = 2.25 - 11 = -8.75$$

$$g \circ f(x) = 5x^2 + 11, \quad f(x) = 2x$$

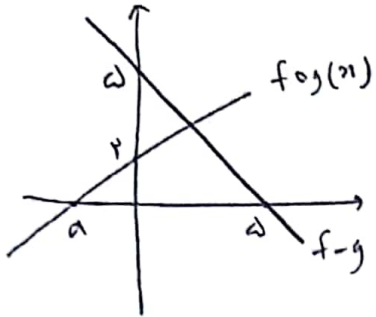
$$g(2x) = 5(2x)^2 + 11 \xrightarrow{x \rightarrow \frac{n}{2}} g(n) = \frac{5n^2}{4} + 11 \xrightarrow{x \rightarrow n-v} g(n-v) = \frac{5(n-v)^2}{4} + 11 = \frac{5n^2 - 10nv + 5v^2}{4} + 11$$

$$\text{کمترین مقدار} \rightarrow g\left(\frac{-b}{2a}\right) = \min_{x \in \mathbb{R}} \rightarrow \frac{-b}{2a} = \frac{-(-10)}{5} = 2 \rightarrow g(2) = \frac{5 \times 4 - 10 \times 2 + 5 \times 4}{4} + 11 = 11$$

$$f(x) = 3x - 1, \quad f \circ g(x) = 3x^2 - 4x - 1$$

$$f(g(x)) = 3x^2 - 4x - 1 \rightarrow 3g(x) - 1 = 3x^2 - 4x - 1 \rightarrow g(x) = x^2 - x + 1 = (x-1)^2$$

$$g(f(x)) = (3x - 1)^2 = (3x - 1)^2 = 9x^2 + 1 - 6x \quad \checkmark \quad \textcircled{y}$$



$$3g(x) = 3f(x) - 1$$

$$(f-g)(x) = ax + b \begin{cases} (f-g)(1) = 1 \rightarrow b = 1 \\ (f-g)(0) = 0 \rightarrow a + 1 = 0 \rightarrow a = -1 \end{cases}$$

$$\rightarrow (f-g)(x) = -x + 1 \rightarrow f(x) - g(x) = -x + 1 \quad \textcircled{1}$$

$$\text{نفس: } 3g(x) - 3f(x) = -1 \quad \textcircled{2}$$

$$\textcircled{1}, \textcircled{2} \rightarrow \begin{cases} f(x) - g(x) = -x + 1 \\ 3g(x) - 3f(x) = -1 \end{cases}$$

$$\rightarrow f(x) = 3x + 1, \quad g(x) = 3x - 1 \quad \textcircled{3} \quad \textcircled{y}$$

$$f \circ g(x) = f(g(x)) \xrightarrow{\textcircled{3}} 3(3x - 1) + 1 = 9x - 2 \Rightarrow f \circ g(x) = 9x - 2$$

$$9x - 2 = 0 \rightarrow 9x = 2 \rightarrow x = \frac{2}{9} = \frac{1}{3} \quad \checkmark$$

$$f(x) = 9^x, \quad g(x) = \log_9 x \quad D_{f \circ g} = x > 0 \quad \textcircled{1} \quad / \quad D_{g \circ f} = 9^x > 0 \rightarrow \mathbb{R} \quad \textcircled{2}$$

$$f \circ g(x) \text{ و } g \circ f(x) \rightarrow 9^{\log_9 x} \text{ و } \log_9 9^x \rightarrow x^{\log_9 9^x} \text{ و } \log_9 9^{x^2} \rightarrow x^x \text{ و } x^2$$

$$\rightarrow x^x - x^2 \text{ و } \rightarrow x(x - x) \text{ و } \rightarrow 0 \text{ و } x \text{ و } 2 \quad \textcircled{1}, \textcircled{2} \rightarrow 0 \text{ و } x \text{ و } 2$$

1, 2, 0

یک عدد صحیح (1) ← 3 عدد صحیح

$$f\left(\frac{n^r - r}{n}\right) = n^r + n - r - \frac{r}{n} + \frac{f}{n^r} = n^r + \frac{f}{n^r} + n - \frac{r}{n} - r = \left(n - \frac{r}{n}\right)^r + r + n - \frac{r}{n} - r$$

$$= \left(n - \frac{r}{n}\right)^r + \left(n - \frac{r}{n}\right) \xrightarrow{n - \frac{r}{n} = t} f(t) = t^r + t \rightarrow f(n) = n^r + n$$

(2)

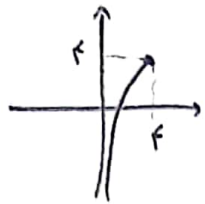
$$f(x) = \log_y x, \quad g(x) = -x^r + \Lambda x$$

$$D_{f \circ g} = A = \{x \in D_g \mid g(x) \in D_f\} = \{-x^r + \Lambda x > 0\} \rightarrow -x^r(\hat{x} - \Lambda) > 0 \rightarrow -x < \Lambda$$

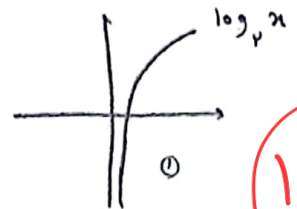
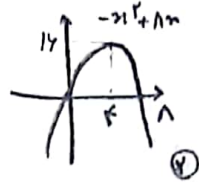
$$R_{f \circ g} = B = f(g(x)) = \log_y (-x^r + \Lambda x)$$

$$\min_{x=0} \rightarrow 0 \quad \lim_{x \rightarrow 0^+} (-\infty)$$

$$\max_{x=f} \rightarrow \log_y 17 = r$$



$$R_{f \circ g} : (-\infty, r]$$



(1, N8)

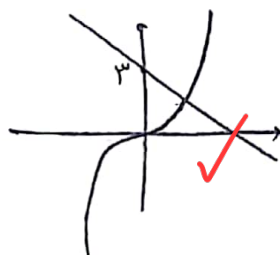
$$A \cap B : ([0, r]) \rightarrow \text{جوابه } [0, r]$$

$$f(f(x)) = 1 - 3f^r(x), \quad g(x) = x - 1, \quad h(x) = x^r + rx + 1$$

$$f(f(x)) = 1 - 3f^r(x) \rightarrow f(x) = 1 - 3x^r$$

$$h(g(x)) = f(x) \rightarrow (x-1)^r + r(x-1) + 1 = 1 - 3x^r \rightarrow x^r + rx - r = -$$

$$\rightarrow x^r = -rx + r$$



يکديگر

(3)

$$f(x) = x - \frac{f}{x}, \quad g(f(x)) = ax^r + rx, \quad g(r) = \Lambda$$

(1)

$$g\left(x - \frac{f}{x}\right) = ax^r + rx, \quad x - \frac{f}{x} = r \rightarrow x = r, \quad x = -1$$

$$g(r) = \Lambda = a(r)^r + rx \rightarrow rfa + \Lambda = \Lambda \rightarrow a = 0$$

$$x = -1 \rightarrow -a - r = \Lambda \rightarrow a = -1$$