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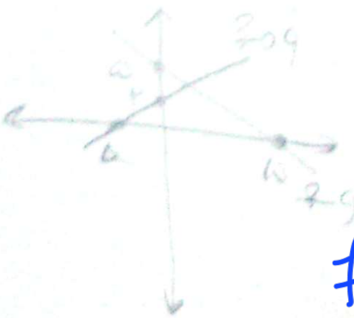
نام و نام خانوادگی .... [illegible] ...... پاسخنامه تشریحی تکلیف شماره ۸ ...... کلاس ....

	$g(x)=\sqrt{-x^2+4x-4}$ $\quad f(x)=\sqrt{\log_2^{(x-1)}}$ $\quad D_{gof(x)}=\{5\} \to \{17\}$ $gof(x)=g(f(x))$ $\log_2^{(x-1)}=4 \Rightarrow 2^4=x-1 \Rightarrow 16=x-1$, $x=5$ → $x=17$ $x-1>0 \Rightarrow x>1$ ✓ این عبارت همواره منفی است — پس فقط ۲ $-x^2+4x-4\geq 0$ ⇒ و به ازای ۲ صفر می‌شود ⇒ در این عبارت صفر می‌شود (1)
۲	(0) $g(x)=x^3-3x+8$ $\quad f(x)=2x-5$ $fog(x)=f(g(x))=2(x^3-3x+8)-5=2x^3-6x+16-5=2x^3-6x+11$ → $2x^2-6x+11$ $gof(x)=g(f(x))=(2x-5)^3-3(2x-5)+8=8x^3-60x^2+...x-102$ → $4x^2-26x+28$ $fog(x)=gof(x) \Rightarrow 6x^3-60x^2+...x-113=0$ → $2x^2-20x+37=0$ → $\alpha^2+\beta^2=S^2-2P=100-37=63$
۳	$f(x)=2x$ $\quad gof(x)=5x^2+11$ $g(f(x))=5x^2+11 \Rightarrow g(2x)=5x^2+11 \Rightarrow 2x=t \Rightarrow x=\frac{t}{2}$ $g(t)=5(\frac{t}{2})^2+11 \Rightarrow g(t)=\frac{5t^2}{4}+11 \Rightarrow g(x)=\frac{5}{4}x^2+11$ $g(x-7) \Rightarrow \frac{5}{4}(x-7)^2+11 \overset{x=7}{\Rightarrow} \frac{5}{4}\times(x^2-14x+49)+11=11$ ✓ (2) $7^2-14\times7+49=0$
۴	(0.5) $fog(x)=3x^2-6x-1$ $\quad f(x)=3x-4$ $f(g(x))=3x^2-6x-1 \Rightarrow 3(g(x))-4=3x^2-6x-1 \Rightarrow 3(g(x))=3x^2-6x-5$ $g(x)=x^2-2x-\frac{5}{3}$ → $g(x)=x^2-2x+1$ $gof(x)=g(f(x))=g(3x-4)=(3x-4)^2-2(3x-4)-\frac{5}{3}=9x^2-24x+16-6x+8-\frac{5}{3}$ $gof(x)=9x^2-30x+24+\frac{5}{3}$ → جواب نهایی $gof(x)=9x^2-30x+25$
۵	 $fog(a)=0 \Rightarrow f(g(a))=0$ (0) $f(0)-g(0)=5 \Rightarrow f(0)=g(0)+5$ $f(x)=2x+4$ $\quad g(x)=3x-1$ $fog(x)=6x+2$ $\quad fog(x)=y=\frac{2}{a}x-2$

$6x+2=\frac{2}{a}x-2 \to a=-\frac{1}{3}$

$f(x) = 9^x \quad g(x) = \log_3^x$

$f \circ g(x) \Rightarrow f(\log_3^x) = 9^{\log_3^x} \Rightarrow x^{\log_3^9} = x^2$

$g \circ f(x) \Rightarrow g(9^x) = \log_3^{9^x} = \log_3^{3^{2x}} = 2x \log_3^3 = 2x$

$f \circ g(x) \leq g \circ f(x) \Rightarrow x^2 \leq 2x \rightarrow [0, 2]$ ✓ (۴)

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$f\left(\frac{x^2-2}{x}\right) = x^2 + x - 4 - \frac{2}{x} + \frac{4}{x^2}$ (۵)

$f\left(x - \frac{2}{x}\right) = \left(x^2 + \frac{4}{x^2} - 4\right) + \left(x - \frac{2}{x}\right) = \left(x - \frac{2}{x}\right)^2 + \left(x - \frac{2}{x}\right)$

$f(x) = x^2 + x$

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$f \circ g(x) = f(g(x)) \Rightarrow f(8x - x^2) = \log_2^x$

$8x - x^2 > 0 \rightarrow x(8-x) > 0 \rightarrow$ (− 0 + 0 −) $\Rightarrow D_{f \circ g} = (0, 8) = A$ (۱)

$R_{f \circ g} = (-\infty, 3) = B$ $\quad A \cap B = (0, 3)$

$B = (-\infty, 4] \quad A \cap B = (0, 4]$ ← عضو صحیح

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(۱۰) $h(g(x)) \Rightarrow h(x-1) = x^2 + 2x + 1 \xrightarrow[x = t+1]{x-1=t} h(t) = (t+1)^2 + 2(t+1) + 1$

$h(t) = t^2 + 2t^2 + 5t + 4$

$h(x) = x^3 + 2x^2 + 5x + 4$

$h(g(x)) = x^3 + 12x^2 + 5x + 4$

$f(x) = 1 - 12x^2$

$h(g(x)) = x^3 - 12x^2 + 5x - 2$

$h(g(x)) = f(x)$

$\hookrightarrow x^3 = -5x + 3 \rightarrow$ یک ریشه

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$g(x^2) = 8 \quad g \circ f(x) = ax^2 + 2x \quad f(x) = x - \frac{4}{2x}$ (۱)

$g(f(x)) = a x^2 + 2x$

$\rightarrow x = -1$

$f(x) = 3 \Rightarrow x - \frac{4}{x} = 3 \Rightarrow 2x = 4 \Rightarrow 4 - \frac{4}{4} = 3$

$a(4)^2 + 2(4) = 8 \Rightarrow 16a + 8 = 8 \Rightarrow \boxed{a = 0}$

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$x = -1 \rightarrow -a - 2 = 8 \rightarrow a = -10$