۱۹,۷۵

سامان نجفی

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۱) $D_{g\circ f}=\{x \mid x\in D_f , f\in D_g\}$

$\Rightarrow D_f \Rightarrow$ I) $x-1>0 \Rightarrow x>1$ II) $\log_2^{x-1}\geq 0 \Rightarrow x-1\geq 1 \Rightarrow x\geq 2 \Rightarrow D_f=[2,+\infty)$

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$D_g \Rightarrow -x^2+4x-4 \geq 0 \Rightarrow -(x-2)^2\geq 0 \Rightarrow x=2 \Rightarrow D_g=\{2\}$

$\Rightarrow D_{f\circ g}$ $\sqrt{\log_2^{x-1}}=2 \Rightarrow \log_2^{x-1}=4 \Rightarrow x-1=16 \Rightarrow x=17 \Rightarrow \boxed{D_{f\circ g}=\{17\}}$

۲) $f\circ g(x)=f(g(x))=f(x^2-3x+8)=2(x^2-3x+8)-5=2x^2-6x+11=f\circ g(x)$

$g\circ f(x)=g(f(x))=g(2x-5)=(2x-5)^2-3(2x-5)+8=4x^2-20x+25-6x+15+8$

$=4x^2-26x+48$

$\Rightarrow f\circ g(x)=g\circ f(x) \Rightarrow 2x^2-6x+11=4x^2-26x+48 \Rightarrow 2x^2-20x+37=0$

$S=-\frac{b}{a}=10 \quad , P=\frac{37}{2} \Rightarrow \alpha^2+\beta^2=S^2-2P \Rightarrow 100-2\times\frac{37}{2}=\boxed{63}$

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۳) $g\circ f(x)=g(f(x)) ; g(2x)=5(x)^2+11 \Rightarrow g(x)=5(\frac{x}{2})^2+11=\frac{5x^2}{4}+11$

$g(x)=\frac{5x^2}{4}+11 \Rightarrow g(x-7)=\frac{5(x-7)^2}{4}+11 \Rightarrow \frac{5(x^2-14x+49)+44}{4}$

$\Rightarrow \frac{5x^2-70x+289}{4} \Rightarrow \frac{5}{4}x^2-\frac{70}{4}x+\frac{289}{4}=g(x-7)$

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$\text{Min}(g(x-7)) \Rightarrow x_S=\frac{\frac{70}{4}}{\frac{5}{2}}=7 \Rightarrow y_S=\frac{5}{4}\times 49-\frac{70}{4}\times 7+\frac{289}{4}=\frac{44}{4}=\boxed{11}$

۴) $f\circ g(x)=f(g(x))=3g-4=3x^2-6x-1 \Rightarrow 3g=3x^2-6x+3 \Rightarrow g=x^2-2x+1$

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$\boxed{g\circ f(x)=g(f(x))=g(3x-4)=(3x-4-1)^2=(3x-5)^2=9x^2-30x+25=g\circ f(x)}$

۵) $f-g=-x+5 \quad , 2g=3f-14 \Rightarrow g=\frac{3}{2}f-7$

$\Rightarrow f-g \xrightarrow{g=\frac{3}{2}f-7} f-\frac{3}{2}f+7=-x+5 \Rightarrow f=2x+4 \Rightarrow g(x)=3x-1$

$f\circ g(x)=f(g(x))=f(3x-1)=2(3x-1)+4=6x-2+4=6x+2=f\circ g(x)$

$\Rightarrow 6a+2=0 \Rightarrow 6a=-2 \Rightarrow \boxed{a=-\frac{1}{3}}$

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⑥ $g\circ f(x) = g(f(x)) = g(9^x) = \log_3 9^x = \log_3 3^{2x} = 2x$

$f\circ g(x) = f(g(x)) = f(\log_3^x) = 9^{\log_3^x} = x^{\log_3^9} = x^2 \quad (x>0)$

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$f\circ g(x) \leq g\circ f(x) \Rightarrow x^2 \leq 2x \Rightarrow x^2-2x \leq 0 \Rightarrow \begin{array}{ccccc} & 0 & & 2 & \\ + & 0 & - & 0 & + \end{array} \Rightarrow \boxed{x\in(0,2]}$

صفر نمی‌تواند باشد زیرا در دامنه تابع fog قرار ندارد

⑦ $f(x-\frac{2}{x}) = \underbrace{x^2-4+\frac{4}{x^2}}_{(x-\frac{2}{x})^2} + x - \frac{2}{x} \xrightarrow{x-\frac{2}{x}=t} f(t) = t^2+t$

$\Rightarrow f(x) = x^2+x$

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⑧ $A = D_{fog} = \{x \mid x\in D_g ,\ g(x)\in D_f\}$

$\Rightarrow D_g = \mathbb{R} ,\ D_f = (0,+\infty) \Rightarrow g(x)\in D_f \Rightarrow 8x - x^2 > 0 \quad \begin{array}{ccccc} & 0 & & 8 & \\ - & 0 & + & 0 & - \end{array} \Rightarrow x\in(0,8)$

$\Rightarrow A = (0,8)$

$R_{fog} = \{(x,y) \mid x\in R_g \cap D_f\} \Rightarrow R_g \Rightarrow \frac{-b}{2a} = x_s = \frac{-8}{-2} = 4 \Rightarrow y_s = -16+32 = 16$

$\Rightarrow R_g = (-\infty, 16]$

$\Rightarrow R_g \cap D_f = (-\infty,16] \cap (0,+\infty) = (0,16] \Rightarrow$ تابع f اکیداً صعودی است و برای بدست آوردن برد آن کافی است ۲ سر دامنه را قرار دهیم

$\Rightarrow R_{fog} = (-\infty, 4] = B$

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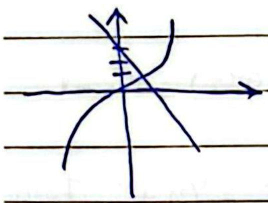
$\Rightarrow A\cap B = (-\infty,4] \cap (0,8) = \boxed{(0,4]}$ → ۴ عدد صحیح

⑨ $f\circ f(x) = f(f(x)) = 1-3f^2(x) \xrightarrow{f(x)=t} f(t) = 1-3t^2 \Rightarrow f(x) = 1-3x^2$

$h(g(x)) = h(x-1) = (x-1)^3 + 2(x-1) + 1 = x^3 - 3x^2 + 3x - 1 + 2x - 2 + 1$

$\Rightarrow x^3 - 3x^2 + 5x - 2 = 1 - 3x^2 \Rightarrow x^3 + 5x = 3 \Rightarrow x^3 = -5x + 3$



$\boxed{\text{۱ ریشه دارد}}$

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⑩ $g\circ f(x) = g(f(x)) = g(x-\frac{4}{x}) = ax^3 + 2x$

$x - \frac{4}{x} = 3 \xrightarrow{\times x} x^2 - 3x - 4 = 0 \Rightarrow \begin{cases} x=-1 \\ x=4 \end{cases}$

I) $x=-1 \Rightarrow g\circ f(-1) = g(3) = 8 = -a-2 \Rightarrow 10 = -a \Rightarrow \boxed{a=-10}$

II) $x=4 \Rightarrow g\circ f(4) = g(3) = 8 = 64a + 8 = 8 \Rightarrow \boxed{a=0}$

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