

الف) $\rightarrow = \left(\sqrt{\frac{x}{9} - \left[\frac{x}{9} \right]} \right)^2 = 4 \left(\frac{x}{9} - \left[\frac{x}{9} \right] \right)$ $\rightarrow y = \sin x + 4$
 $0 - [0] \in [0, 1)$ $\in [0, 1)$ $\rightarrow 3 \leq y \leq 5$ $\rightarrow R = [0, 4]$ $\rightarrow R = [10^3, 10^5]$

دوسر تابع را می دهیم سپس عمل را اعمال می کنیم:
 $\rightarrow \frac{x^2 - 4}{x^2 - 9} \rightarrow 0 \rightarrow \frac{4}{9}$
 $\rightarrow R = (-\infty, \frac{4}{9}] \cup (1, +\infty) \rightarrow R' = \sqrt{(-\infty, \frac{4}{9}] \cup (1, +\infty)}$
 $\rightarrow R' = [0, \frac{2}{3}] \cup (1, +\infty) = [a, b] \cup (c, +\infty) \rightarrow \begin{cases} a=0 \\ b=\frac{2}{3} \\ c=1 \end{cases} \rightarrow a+b+c = 0 + \frac{2}{3} + 1 = 1\frac{2}{3} = \frac{5}{3}$

الف) $\rightarrow = \left(\sqrt{x + \frac{1}{x}} + 1 \right)^2 - 1 \rightarrow \min = (\sqrt{2} + 1)^2 - 1$ $\rightarrow -\sin^2 x + 2\sin x + 3 =$
 $\rightarrow x + \frac{1}{x} \geq 0 \rightarrow x > 0 \rightarrow x + \frac{1}{x} \geq 2$
 $3 + 2\sqrt{2} - 1 = 2 + 2\sqrt{2} \rightarrow R = [2 + 2\sqrt{2}, +\infty)$
 $\rightarrow -(\sin^2 x - 2\sin x + 1 - 4) = -((\sin x - 1)^2 - 4)$
 $\rightarrow = -(\sin x - 1)^2 + 4 \Rightarrow -4 \leq \sin x - 1 \leq 0$
 $\rightarrow 0 \leq (\sin x - 1)^2 \leq 4 \rightarrow -4 \leq -(\sin x - 1)^2 \leq 0$
 $\rightarrow 0 \leq -(\sin x - 1)^2 + 4 \leq 4 \rightarrow R = [0, 4]$

$\rightarrow f(x) = -\frac{1}{3}x + 10$ و $D_f = \{x | x \in \mathbb{N}, x < 10\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

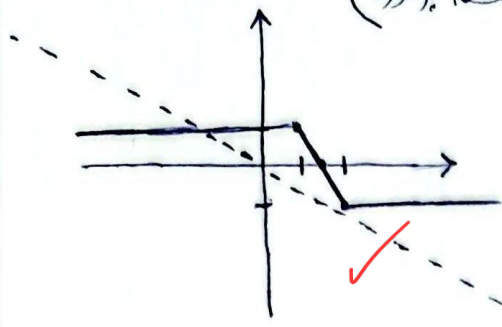
$\rightarrow S_R = -\frac{1}{3}(1+2+3+\dots+9) + 9(10) = -\frac{45}{3} + 90 = -15 + 90 = 75$

$f(x) = \sqrt{ax^2 + bx + c}$ $\rightarrow \begin{cases} a=0 \\ b=1 \\ c=-2 \end{cases}$ و $x=2 \rightarrow 2b+c=0$ (I)

$f(2)=1 \rightarrow 2b+c=1$ (II) $\rightarrow \begin{cases} b=1 \\ c=-2 \end{cases} \rightarrow R = [2, +\infty)$

$g(x) = \sqrt{bx^2 + ax - 2c} = \sqrt{x^2 + 4} \rightarrow x_{\min} = 0 \rightarrow y_{\min} = 2$

رسم شکل: $|x-2|-|x-1|=kx$ (دو جواب = دو نقطه برخورد)



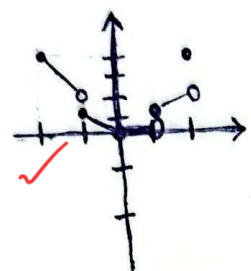
$$\Rightarrow (0,0), (2,-1) \Rightarrow k = \frac{\Delta y}{\Delta x} = \frac{0-(-1)}{0-2} = -\frac{1}{2}$$

$$k = -\frac{1}{2}$$

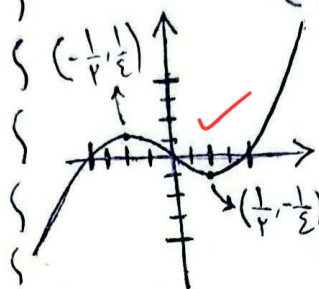
الف) $x[x]$

$$= \begin{cases} -2x & -2 \leq x < -1 \\ -x & -1 \leq x < 0 \\ 0 & 0 \leq x < 1 \\ x & 1 \leq x < 2 \\ 2x & x = 2 \end{cases}$$

$$R = [0, 4] - \{2\}$$



$$x|x| - x = \begin{cases} x^2 - x & x \geq 0 \\ -x^2 - x & x < 0 \end{cases}$$

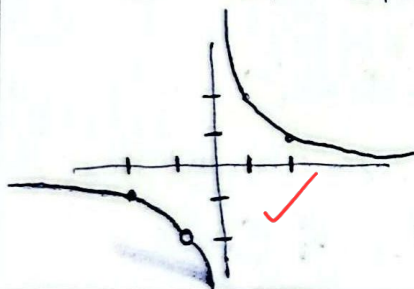


$$\Rightarrow R = \mathbb{R}$$

$$R = [-2, 2]$$

با توجه به اینکه بین سوال

$$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x} = \frac{2x+2}{x^2+1} = \frac{2(x+1)}{x(x+1)} = \frac{2}{x}$$



$$D = \mathbb{R} - \{0, -1\}$$

$$a+b = 0+(-2) = -2$$

$$\Rightarrow R = \mathbb{R} - \{a, b\} = \mathbb{R} - \{-2, 0\}$$

$$f(x) = \frac{(\sqrt{x}-1)(\sqrt{x}-3)}{(\sqrt{x}-1)^2} = \frac{\sqrt{x}-3}{\sqrt{x}-1}$$

$$D = [0, +\infty) - \{1\}$$

با توجه به دامنه و مقادیر بدست آمده

$$R = (1, 3] - \{y | (1, y)\}$$

$$R_f = (-\infty, 1) \cup [3, +\infty)$$

در دامنه نسبت پس در برد برای مقدار آن دایره توخالی می گذاریم

$$f(x) = \frac{x^3+2x^2-x-2}{x^2-1} = \frac{x^2(x+2)-(x+2)}{x^2-1} = (x+2) \frac{x^2-1}{x^2-1} = x+2$$

$$\Rightarrow \begin{matrix} x=1 \rightarrow y=3 \\ x=-1 \rightarrow y=1 \end{matrix}$$

$$\Rightarrow S = 3+1 = 4$$