

$$y^2 - y = x^2 + x + 2 \rightarrow x^2 + x(1-y) + (y+2) = 0 \rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow \Delta \geq 0$$

$$\Rightarrow (1-y)^2 - 4(y+2) \geq 0 \Rightarrow 1 + y^2 - 2y - 4y - 8 \geq 0 \rightarrow y^2 - 6y - 7 \geq 0$$

$$\Rightarrow (y-7)(y+1) \geq 0 \quad \frac{-1}{+b} \quad \frac{7}{-b+} \rightarrow R_f = (-\infty, -1] \cup [7, +\infty)$$

الف) $\alpha > 0 \rightarrow \begin{cases} x_{\min} = \frac{-b}{2\alpha} = \frac{5}{2} \\ y_{\min} = -\frac{31}{19} = -\frac{19}{1} \end{cases} \rightarrow R_f = [-\frac{19}{1}, +\infty)$

ب) $\alpha < 0 \rightarrow \begin{cases} x_{\max} = \frac{-b}{2\alpha} = \frac{-9}{-2} = \frac{9}{2} \\ y_{\max} = 4 \end{cases} \rightarrow R_f = \sqrt{[4, +\infty)} = [2, +\infty)$

ج) $R_f = [0, +\infty)$

د) $R_f = (-\infty, +\infty)$

الف) $R_f = \mathbb{R} - \{\frac{3}{2}\}$

ب) $R_f = [0, +\infty) - \{2\} \rightarrow \sqrt{4}$

ج) $\frac{x^2+3+1}{\sqrt{x^2+3}} = \sqrt{x^2+3} + \frac{1}{\sqrt{x^2+3}} \rightarrow R_f = [2, +\infty)$

د) $\frac{x^2+4}{+} + \frac{1}{x^2+4} - 4 \rightarrow R_f = [4-4, +\infty) = [0, +\infty)$

$x^2 = 0 \rightarrow y = -4$ منجم صفری شود $(-\infty, -4] \cup (3, +\infty)$

$x^2 = +\infty \rightarrow y = 3$

$\sin x = 1 \rightarrow y = \frac{1}{4}$ منجم صفری شود $[-\frac{3}{4}, \frac{1}{4}]$

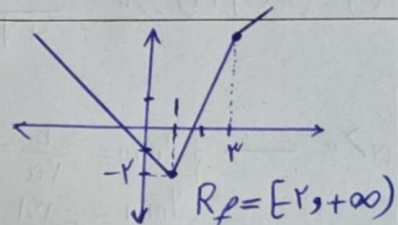
$\sin x = -1 \rightarrow y = -\frac{3}{4}$

$$h(x) = x^2 - x \xrightarrow{\alpha > 0} \begin{cases} x_{\min} = -\frac{b}{2a} = \frac{1}{2} \\ y_{\min} = -\frac{1}{4} \end{cases} \Rightarrow f(x) = \frac{h(x) + \frac{1}{4}}{h(x) + 1}$$

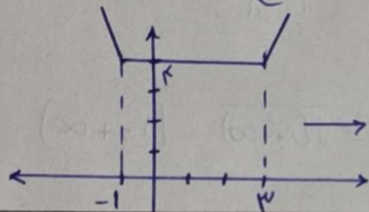
$$h(x) = -\frac{1}{4} \rightarrow f(x) = 0$$

$$h(x) = +\infty \rightarrow f(x) = 1 \xrightarrow[\text{منفی شود}]{\text{بیشتر می شود}} R_f = [0, 1)$$

الف) $f(x) = \begin{cases} 2x-2-x+3 & x \geq 3 \\ 2x-2+x-3 & -1 \leq x < 3 \\ 2-2x+x-3 & x < -1 \end{cases} \rightarrow \begin{cases} x+1 & x \geq 3 \\ 3x-5 & -1 \leq x < 3 \\ -x-1 & x < -1 \end{cases}$



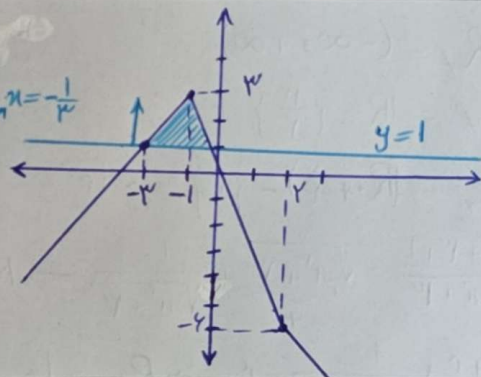
ب) تابع گدازی:



$$\rightarrow R_f = [2, +\infty)$$

$$\begin{cases} 2x-2 & x \geq 3 \\ 2 & -1 \leq x < 3 \\ -2x+2 & x < -1 \end{cases}$$

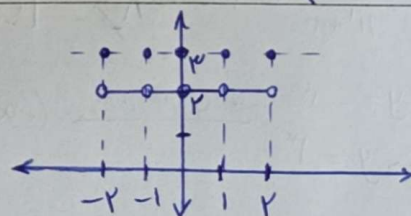
$$\begin{cases} x-2-2x-2 & 2 \leq x \\ 2-x-2x-2 & -1 \leq x < 2 \\ 2-x+2x+2 & x < -1 \end{cases} \rightarrow \begin{cases} -x-2 & 2 \leq x \\ -3x & -1 \leq x < 2 \\ x+2 & x < -1 \end{cases}$$



$$\text{محدوده جواب: } [-3, -\frac{1}{2}]$$

الف) $y = [x] + [-x] + 3$

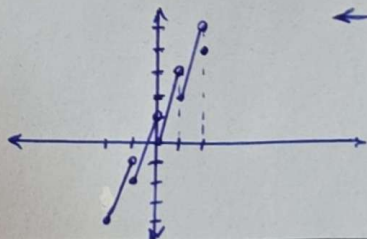
$$\Rightarrow R_f = \{2, 3\}$$



ب) $y = 2x + x - [x]$

$$x=1 \rightarrow y=2 \quad x=0 \rightarrow y=0$$

$$x=2 \rightarrow y=4$$



$$R_f = \mathbb{R}$$

$$-1 \leq \sin x \leq 1 \rightarrow \sin x = 1 \Rightarrow y = 1 - 1 + 1 = 1$$

$$\rightarrow \sin x = -1 \Rightarrow y = 1 + 1 + 1 = 3 \rightarrow \text{بیشترین}$$

$$-\frac{b}{2a} = \frac{1}{2} \rightarrow \sin x = \frac{1}{2} \Rightarrow y = \frac{1}{2} - \frac{1}{2} + 1 = 1 \rightarrow \text{کمترین} \rightarrow R_f = [\frac{3}{2}, 3]$$

الف)

$$0 \leq \sin^2 x \leq 1 \rightarrow \sin^2 x = 1 \Rightarrow y = 1 + 1 + 1 = 3 \rightarrow \text{بیشترین}$$

$$\rightarrow \sin^2 x = 0 \Rightarrow y = 0 + 0 + 1 = 1 \rightarrow \text{کمترین} \rightarrow R_f = [1, 3]$$

ب)

$$-\frac{b}{2a} = -\frac{1}{2} \rightarrow \sin^2 x = -\frac{1}{2} \times \text{غیرممکن}$$