

$$x = \frac{z^2 + z + 2}{z - 1} \rightarrow yz - x = z^2 + z + 2 \rightarrow z^2 + z(1 - y) + 2 + y = 0$$

$$\rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow x = \frac{-(1-y) \pm \sqrt{(1-y)^2 - 4(2+y)}}{2} \rightarrow \Delta \geq 0$$

$$\rightarrow (1-y)^2 - 4(2+y) \geq 0 \rightarrow 1 + y^2 - 2y - 8 - 4y - 4 \geq 0 \rightarrow y^2 - 6y - 11 \geq 0$$

$$\rightarrow (y-7)(y+1) \geq 0 \rightarrow R_f = (-\infty, -1] \cup [7, +\infty)$$

$$\text{الف) } x = 2z^2 - 2z + 1 \quad \text{ب) } y = \sqrt{-z^2 + 4z - 5} \quad z_{\max} = \frac{-b}{2a} = \frac{-2}{-4} = \frac{1}{2}$$

چون تابع در دو حالت بررسی می شود:

$$\frac{-b}{2a} = \frac{-(-2)}{4} = \frac{1}{2} \Rightarrow x_{\min} = \frac{1}{2}$$

$$y_{\min} = \frac{-1}{2} = -\frac{1}{2}$$

$$\rightarrow R_x = [-\frac{1}{2}, +\infty)$$

چون تابع در دو حالت بررسی می شود:

$$y_{\max} = 4$$

$$R_y = [-\infty, 4]$$

برای تابع معده ای که بزرگترین توان آن باشد برابر R است اینجا باید به رادیکال دقت کنیم

$$R_x = \sqrt{1R} = [0, +\infty)$$

$$\Rightarrow \log(z^2 - 4z^2 + 7z + 4) \quad R = (-\infty, +\infty)$$

$$\text{الف) } x = \frac{3x+1}{2x-4} \quad \text{ب) } y = \sqrt{\frac{4x+1}{x-2}}$$

تابع معکوس است و در $x=2$ برابر $R = \{2\}$

$$\rightarrow R_x = R - \{2\}$$

تقریب مقدار دهی می توانیم بکنیم

$$R_x = \sqrt{R - \{2\}} = [0, +\infty) - \{2\}$$

$$\text{ج) } x = \frac{z^2 + 4}{\sqrt{z^2 + 3}} = \frac{z^2 + 3 + 1}{\sqrt{z^2 + 3}} = \frac{z^2 + 3}{\sqrt{z^2 + 3}} + \frac{1}{\sqrt{z^2 + 3}}$$

تقریب مقدار دهی می توانیم بکنیم

$$R_x = \sqrt{\frac{4}{3}} = [\frac{2}{\sqrt{3}}, +\infty)$$

$$y = \frac{3x^2 + 4}{x^2 - 1} \quad \text{روش اول: } x = \frac{3y^2 + 4}{y^2 - 1} \rightarrow xy^2 - x = 3y^2 + 4 \rightarrow 2y^2 - 3y^2 = 4 + x$$

$$\rightarrow y^2(x-3) = 4+x \rightarrow y^2 = \frac{4+x}{x-3} \rightarrow y = \sqrt{\frac{4+x}{x-3}}$$

تعیین علامت

-4	3
$\frac{4+x}{x-3}$	$\frac{4+x}{x-3}$
-	+
+	-
+	+

روش دیگر: $x = \frac{3x^2 + 4}{x^2 - 1}$

چون مخرج می تواند صفر شود و 0 و $+$ و $-$ پس برداریم این دو عدد:

یکبار صفر دهیم $\rightarrow y = -4$

یکبار به بی نهایت $\rightarrow y = \frac{3}{\infty} = 0$

تعیین علامت

$$R_x = (-\infty, -4] \cup (0, +\infty)$$

$$y = \frac{y \sin x - 1}{\sin x + 3} \quad \text{روش اول: } x = \frac{y \sin y - 1}{\sin y + 3} \rightarrow x \sin y + 3x = y \sin y - 1 \rightarrow x \sin y - y \sin y = -1 + 3x$$

$$\rightarrow \sin y(x-1) = -1 + 3x \rightarrow \sin y = \frac{-1+3x}{x-1}$$

تعیین علامت

$$-1 \leq \frac{-1+3x}{x-1} \leq 1$$

چون مخرج را به بی نهایت نزدیک می کنیم

$$R_x = [0, 1] \cap [1, 3] = [1, 3]$$

تعیین علامت

$$R_x = [0, 1] \cup [3, +\infty)$$

یکبار $\rightarrow \frac{2-1}{3} = \frac{1}{3}$

یکبار $\rightarrow \frac{2-1}{-1+3} = \frac{1}{2}$

تعیین علامت

$$R_x = [-\frac{1}{3}, \frac{1}{2}]$$

$$f(x) = \frac{x^2 - x + \frac{1}{4}}{x^2 - x + 1}$$

$$x^2 - x + 1 \neq 0 \rightarrow \Delta = (-1)^2 - 4 \times 1 = -3 \quad \text{ریشه ندارد و } \Delta < 0$$

$$\rightarrow D_f = \mathbb{R}$$

$$\frac{x^2 - x + \frac{1}{4}}{x^2 - x + 1} \quad \frac{x^2 - x + \frac{1}{4}}{x^2 - x + 1} = t \quad \frac{t + \frac{1}{4}}{t + 1} \quad t = -\frac{1}{4} \rightarrow x = 0$$

$$R_y = \left[\frac{1}{5}, 1 \right)$$

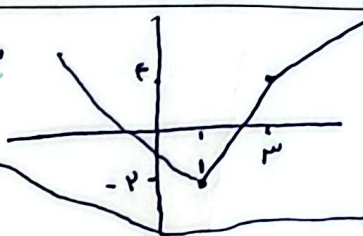
مقدار $t = +\infty$ بیشتر مقدار

چون خارج نمی تواند صفر شود پس بین این دو عدد است

$$الف) y = |2x - 2| - |x - 2|$$

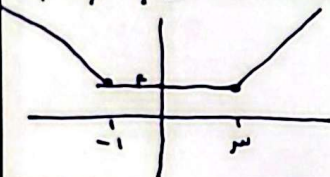
$$بر اساس شکل: R_y = [-2, +\infty)$$

$$\begin{array}{c|c|c} 1 & 2 & 3 \\ \hline 2-2x-2+x & 2x-2-2+x & 2x-2-2+3 \\ \hline -x-1 & x-4 & x+1 \end{array}$$



$$ب) y = |x-2| + |x+1|$$

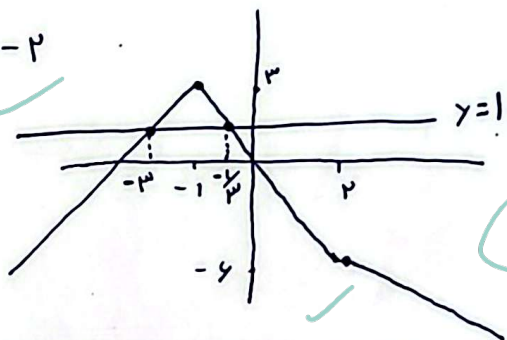
$$\begin{array}{c|c|c} -1 & 2 & 3 \\ \hline 3-x-2-1 & 2-2+x+1 & x-2+x+1 \\ \hline -x & x & x-1 \end{array}$$



$$R_y = [3, +\infty)$$

$$|x-2| - |2x+2|$$

$$\begin{array}{c|c|c} -1 & 2 & 3 \\ \hline 2-x+2x+2 & 2-x-2x-2 & x-2-2x-2 \\ \hline x+4 & -3x-4 & -x-4 \end{array}$$



$$\text{دامنه جواب} \rightarrow (-\infty, -3] \cup \left[-\frac{1}{3}, +\infty\right)$$

$$الف) y = [x] + [3-x] = [x] + [-x] + 3$$

$$ب) y = 3x - [x] \quad x \in [-2, 2]$$

$$-2 \leq x < -1 \rightarrow [x] = -2 \rightarrow y = 3x + 2$$

$$-1 \leq x < 0 \rightarrow [x] = -1 \rightarrow y = 3x + 1$$

$$0 \leq x < 1 \rightarrow [x] = 0 \rightarrow y = 3x$$

$$1 \leq x < 2 \rightarrow [x] = 1 \rightarrow y = 3x - 1$$

$$2 \leq x \leq 2 \rightarrow [x] = 2 \rightarrow y = 3x - 2$$

$$3 \leq x \leq 3 \rightarrow [x] = 3 \rightarrow y = 3x - 3$$

$$4 \leq x \leq 4 \rightarrow [x] = 4 \rightarrow y = 3x - 4$$

$$5 \leq x \leq 5 \rightarrow [x] = 5 \rightarrow y = 3x - 5$$

$$6 \leq x \leq 6 \rightarrow [x] = 6 \rightarrow y = 3x - 6$$

$$7 \leq x \leq 7 \rightarrow [x] = 7 \rightarrow y = 3x - 7$$

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$$9 \leq x \leq 9 \rightarrow [x] = 9 \rightarrow y = 3x - 9$$

$$10 \leq x \leq 10 \rightarrow [x] = 10 \rightarrow y = 3x - 10$$

$$11 \leq x \leq 11 \rightarrow [x] = 11 \rightarrow y = 3x - 11$$

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