

$$y = \frac{x^2 + x + 2}{x - 1} \xrightarrow{\text{مضرب و مضروب}} y(x-1) = x^2 + x + 2 \rightarrow x^2 + (1-y)x + 2+y = 0$$

$$\rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow x = \frac{y-1 \pm \sqrt{y^2 - 4y - 1}}{2} \rightarrow \frac{y-1 \pm \sqrt{y^2 - 4y - 1}}{2}$$

$$\xrightarrow{\text{دامنه جابجایی}} \frac{y-1 \pm \sqrt{y^2 - 4y - 1}}{2} \rightarrow y^2 - 4y - 1 \geq 0 \rightarrow \frac{-1}{2} - \frac{1}{2} \rightarrow R_f = (-\infty, -1] \cup [7, +\infty)$$

$$\text{الف} \rightarrow x > 0 \rightarrow \min y \rightarrow \min(y_5) = -\frac{\Delta}{4a} \rightarrow -\frac{b^2 - 4ac}{4a} \rightarrow -\frac{10 - 1}{4} \rightarrow -\frac{9}{4}$$

$$\xrightarrow{\text{الف}} R_f = [-\frac{9}{4}, +\infty) \quad \text{ب} \rightarrow \sqrt{-x^2 + 4x - 5} \rightarrow \sqrt{-(x-2)^2 - 1} \rightarrow \sqrt{4} = 2 \rightarrow R_f = [0, 2]$$

$$\text{ج} \rightarrow \sqrt{x^2 - 3x^2 + 4x + 1} \rightarrow \sqrt{-2x^2 + 4x + 1} \rightarrow R_f = [0, +\infty) \quad \text{د} \rightarrow \log(x^2 - 9x^2 + 14x + 4) \rightarrow R_f = \mathbb{R}$$

$$\text{الف} \rightarrow \frac{3x+1}{x^2-4} \rightarrow \min \rightarrow -\frac{1}{4} \quad \text{ب} \rightarrow \sqrt{\frac{4x+1}{x-2}} \rightarrow \min \rightarrow -\frac{1}{2} \rightarrow R = \mathbb{R} - \{2\}$$

$$\text{ج} \rightarrow \sqrt{\frac{4x+1}{x-2}} \rightarrow \min \rightarrow -\frac{1}{2} \rightarrow R = \mathbb{R} - \{2\}$$

$$\text{د} \rightarrow \sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}} \rightarrow \min \rightarrow \frac{2}{\sqrt{3}} \rightarrow R_f = [\frac{2}{\sqrt{3}}, +\infty)$$

$$\text{الف} \rightarrow yx^2 - y = 4x^2 + 4 \rightarrow (y-4)x^2 = 4+y \rightarrow x^2 = \frac{4+y}{y-4} \rightarrow x = \pm \sqrt{\frac{4+y}{y-4}} \rightarrow R_f = (3, +\infty) \cup (-\infty, -4]$$

$$\text{ب} \rightarrow \min x^2 \rightarrow y = -4 \rightarrow R_f = (-\infty, -4] \cup (3, +\infty) \quad \text{ج} \rightarrow \max x^2 \rightarrow y = 3 \rightarrow R_f = (-\infty, -4] \cup (3, +\infty)$$

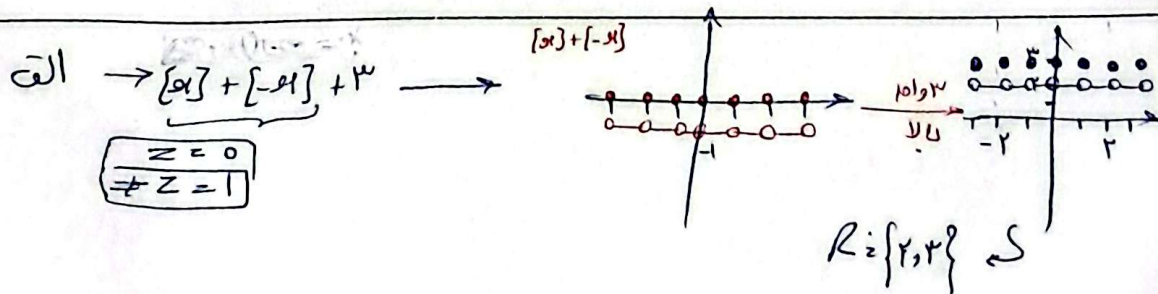
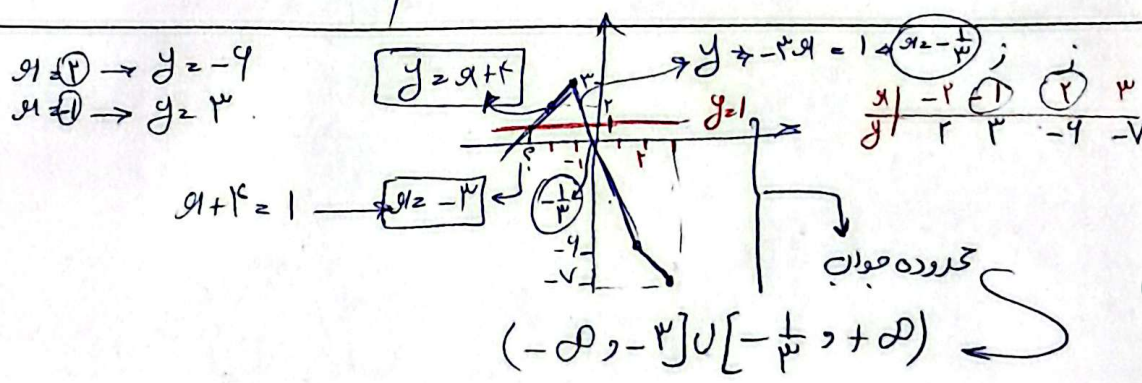
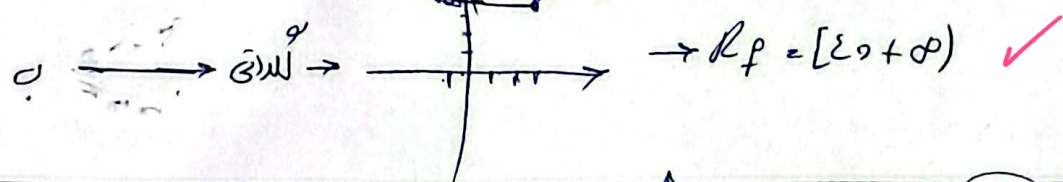
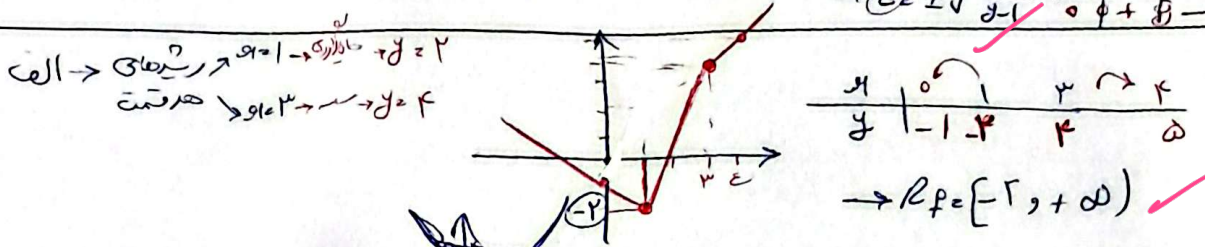
$$\text{الف} \rightarrow y \sin x + 3y = 2 \sin x - 1 \rightarrow (y-2) \sin x = -3y-1 \rightarrow \sin x = \frac{-3y-1}{y-2}$$

$$\rightarrow -1 \leq \frac{-3y-1}{y-2} \leq 1 \rightarrow \text{الف} \rightarrow \frac{-3y-1}{y-2} \leq 1 \rightarrow \frac{-3y-1-y+2}{y-2} \leq 0 \rightarrow \frac{-4y+1}{y-2} \leq 0 \rightarrow R_f = [-\frac{1}{4}, 2)$$

$$\text{ب} \rightarrow \frac{-3y-1}{y-2} \geq -1 \rightarrow \frac{-3y-1+y+2}{y-2} \geq 0 \rightarrow \frac{-2y+1}{y-2} \geq 0 \rightarrow R_f = (-\infty, \frac{1}{2}] \cup (2, +\infty)$$

الف  $\rightarrow x^2 - x + 1 \neq 0 \rightarrow \Delta < 0 \rightarrow$  دایره حلقه  $\rightarrow D_f = R$  ✓

ب  $\rightarrow \frac{x^2 - x + \frac{1}{\varepsilon}}{x^2 - x + 1} \rightarrow \frac{(x - \frac{1}{\varepsilon})^2}{x^2 - x + 1 - \frac{x}{\varepsilon} + \frac{1}{\varepsilon^2}} \rightarrow \frac{z^2}{z^2 + \frac{1}{\varepsilon}} = z^2$   
 $\rightarrow (y - \frac{1}{\varepsilon})^2 = -\frac{1}{\varepsilon}y$   
 $\rightarrow (y - \frac{1}{\varepsilon})^2 = -\frac{1}{\varepsilon}y$  (0, 1) ✓



ب  $\rightarrow$  ✓

$y \rightarrow \sin^2 x - \sin x + 1$

$\begin{cases} \min \sin x (-1) \rightarrow 3 \\ \max \sin x (1) \rightarrow 1 \\ -\frac{b}{2a} (\frac{1}{2}) \rightarrow \frac{5}{4} \end{cases} \rightarrow [\frac{5}{4}, 3]$

$y \rightarrow \sin^2 x + \sin^2 x + 1$

$\begin{cases} \min \sin^2 x \rightarrow 0 \rightarrow 1 \\ \max \sin^2 x + 1 \rightarrow 3 \\ -\frac{b}{2a} \rightarrow -\frac{1}{2} \text{ outside } \sin^2 x \end{cases} \rightarrow [1, 3]$