

$$yx - y = x^2 + x + 1 \Rightarrow x^2 + (1-y)x + y + 1$$

$$\Delta \geq 0 \Rightarrow b^2 - 4ac \geq 0 \Rightarrow (1-y)^2 - 4(y+1) \geq 0 \Rightarrow y^2 - 2y + 1 - 4y - 4 \geq 0$$

$$\Rightarrow y^2 - 4y - 3 \geq 0$$

$$\Rightarrow \frac{-1 \pm \sqrt{1+12}}{2} \Rightarrow Rf = (-\infty, -1] \cup [3, +\infty)$$

$$a) \frac{-b}{2a} = \frac{0}{2} \Rightarrow 1 \left( \frac{0}{2} \right)^2 - 0 \left( \frac{0}{2} \right) + 1 = \frac{-1}{1} \Rightarrow Rf = \left[ \frac{-1}{1}, +\infty \right)$$

$$b) \frac{-b}{2a} = 3 \Rightarrow \sqrt{-1+12-0} = \sqrt{11} = 3 \Rightarrow Rf = [0, 3]$$

$$c) \Rightarrow Rf = [0, +\infty)$$

$$d) \Rightarrow Rf = (-\infty, +\infty)$$

$$a) Rf = R - \left[ \frac{0}{2} \right] \Rightarrow Rf = R - \left[ \frac{0}{2} \right]$$

$$b) Rf \left( \frac{ex+1}{x-1} \right) = [0, +\infty) - \left[ \frac{1}{1} \right] \Rightarrow Rf \sqrt{\frac{ex+1}{x-1}} = \sqrt{[0, +\infty) - 1} \Rightarrow Rf = [0, +\infty) - [1]$$

$$c) Rf \Rightarrow x^2 \rightarrow \frac{f}{13} = \frac{ex}{13} \Rightarrow Rf = \left[ \frac{ex}{13}, +\infty \right)$$

$$d) y + \varepsilon = x^2 + \varepsilon + \frac{1}{x^2 + \varepsilon} \xrightarrow{x \rightarrow \infty} y + \varepsilon = \varepsilon \frac{1}{\varepsilon} \Rightarrow Rf = \left[ \frac{1}{\varepsilon}, +\infty \right)$$

$$yx^2 - y = 3x^2 + \varepsilon \Rightarrow yx^2 - 3x^2 = y + \varepsilon \Rightarrow x^2 = \frac{y + \varepsilon}{y - 3}$$

$$x = \sqrt{\frac{y + \varepsilon}{y - 3}} \Rightarrow Rf = (-\infty, -1] \cup (3, +\infty)$$

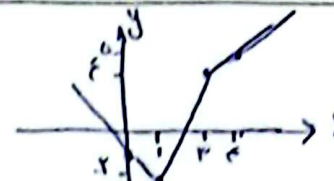
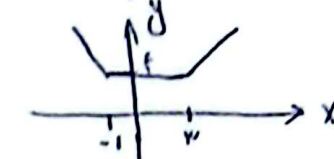
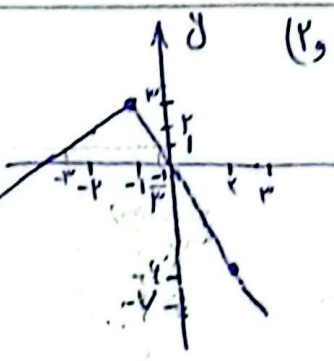
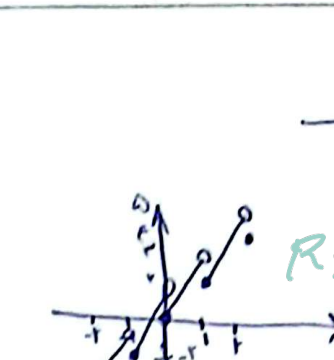
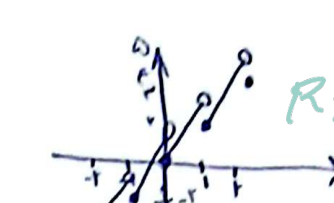
$$x^2 = 0 \rightarrow y = \frac{0 + \varepsilon}{0 - 3} = -\frac{\varepsilon}{3} \rightarrow y \approx 3 \Rightarrow Rf = (-\infty, -1] \cup (3, +\infty)$$

$$y \sin x + 3y = 2 \sin x - 1 \Rightarrow y \sin x - 2 \sin x = -3y - 1 \Rightarrow \sin x = \frac{-3y - 1}{y - 2}$$

$$\Rightarrow -1 \leq \frac{-3y - 1}{y - 2} \leq 1 \Rightarrow Rf = \left[ \frac{-3}{2}, \frac{1}{\varepsilon} \right] \Rightarrow Rf = R$$

$$-1 \leq \sin x \leq 1 \Rightarrow y_1 = \frac{|x| - 1}{1 + 3} = \frac{1}{4}, y_2 = \frac{2x(-1) - 1}{-1 + 3} = \frac{3}{2}$$

$$\Rightarrow Rf = \left[ \frac{-3}{2}, \frac{1}{\varepsilon} \right]$$

|                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                                        |        |
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| $f(x) = \frac{(x - \frac{1}{x})^2}{(x - \frac{1}{x})^2 + \frac{x^2}{x}}$ $(x - \frac{1}{x})^2 + \frac{x^2}{x} \rightarrow \frac{0}{\infty} \rightarrow \frac{0}{\infty} \rightarrow \frac{0}{\infty} \rightarrow \frac{0}{\infty}$ $t = 0 \rightarrow y_1 = 0, t = \infty \rightarrow y_2 = 1$ <p style="color: red;">خرج من 0 و 1</p>                                                                | $(x - \frac{1}{x}) \rightarrow t \Rightarrow f(x) = \frac{t^2}{t^2 + \frac{x^2}{x}}$ $Df = R$ $Rf = (-\infty, 0] \cup (1, +\infty)$ <p style="color: red;"><math>Rf \neq [0, 1]</math></p>                                             | 1, 100 |
| الف) <br>ب)                                                                                                                                                                                                                         | $(0, -1), (1, -2), (2, 4), (3, 5)$<br>$Df = R, Rf = [-2, +\infty)$<br>$Df = R, Rf = [5, +\infty)$                                                                                                                                      | 2      |
|                                                                                                                                                                                                                                                                                                                     | $(2, -4), (3, -7), (-1, 3), (-2, 2)$<br>$\begin{array}{c c c} -1 & 2 & \\ \hline x+1 & -3x & -x-2 \end{array}$ $x+2 > 1 \Rightarrow x > -3, 1 > -3x \Rightarrow \frac{1}{3} > x$ $\hookrightarrow \text{مجال} = R - (\frac{1}{3}, -3)$ | 2      |
| الف) <br>ب)                                                                                                                                                                                                                     | $Rf = \{2, 3\}$<br>كامل تميزي و باهني من<br>$Rf = [-4, 0)$                                                                                                                                                                             | 1      |
| الف) $\sin x$ $\begin{cases} 1 \Rightarrow 1 - 1 + 1 = 1 \\ -1 \Rightarrow 1 - (-1) + 1 = 3 \\ \frac{-b}{2a} = \frac{1}{2} \Rightarrow \frac{1}{2} - \frac{1}{2} + 1 = \frac{3}{2} \end{cases}$<br>ب) $\sin^2 x$ $\begin{cases} 1 \Rightarrow 1 + 1 + 1 = 3 \\ 0 \Rightarrow 0 + 0 + 1 = 1 \\ \frac{-b}{2a} = \frac{-1}{2} \Rightarrow (\frac{-1}{2})^2 + \frac{-1}{2} + 1 = \frac{3}{4} \end{cases}$ | $\Rightarrow Rf = [\frac{3}{2}, 3]$<br>$\Rightarrow Rf = [\frac{3}{4}, 3]$ <p style="color: green;"><math>\hookrightarrow \sin^2 x \neq \frac{3}{4}</math></p>                                                                         | 1, 100 |