

$$y = \frac{x^2 + x + 2}{x - 1} \Rightarrow yx - y = x^2 + x + 2 \Rightarrow x^2 + (1 - y)x + 2 + y = 0 \Delta \geq 0$$

$$(1 - y)^2 - 4(2 + y) \geq 0 \Rightarrow y^2 - 6y - 7 \geq 0 \Rightarrow y \in (-\infty, -1] \cup [7, +\infty)$$

$$\Rightarrow R_{\text{د}} = (-\infty, -1] \cup [7, +\infty)$$

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$y_{\min} = -\frac{\Delta}{4a}$	$y_{\max} = \sqrt{-\frac{\Delta}{4a}}$	$R_{\text{د}} = (-\infty, +\infty)$	$R_{\text{د}} = (-\infty, +\infty)$
$y_{\min} = -\frac{17}{8}$	$y_{\max} = \sqrt{1}$	$R_{\text{د}} = [0, 4]$	$R_{\text{د}} = [0, +\infty)$
$\Rightarrow R_{\text{د}} = [-\frac{17}{8}, +\infty)$			

$$y = \frac{x^2 + 4}{\sqrt{x^2 + 3}} = \frac{x^2 + 3 + 1}{\sqrt{x^2 + 3}} = \sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}}$$

$$\Rightarrow R_{\text{د}} = [\frac{4}{\sqrt{3}}, +\infty)$$

$$y = \frac{x^2 + 1}{x^2 + 4} = \frac{x^2 + 4 - 3}{x^2 + 4} = 1 - \frac{3}{x^2 + 4}$$

$$\Rightarrow R_{\text{د}} = [\frac{1}{4}, 1)$$

$$y = \frac{3x^2 + 4}{x^2 - 1} \xrightarrow{x=0} y = -4 \Rightarrow R_{\text{د}} = (-\infty, -4] \cup (3, +\infty)$$

$$y = 3 \Rightarrow -y - 4 = 0 \Rightarrow y = -4$$

$$y = 3 \Rightarrow (y - 3)x^2 - y - 4 = 0 \Rightarrow (y - 3)x^2 - y - 4 = 0$$

$$\Rightarrow -4(y - 3)(-y - 4) \geq 0 \Rightarrow y = R_{\text{د}} \in (-\infty, -4] \cup (3, +\infty)$$

$$y = \frac{2 \sin x - 1}{\sin x + 3}$$

$$\sin x = 1 \Rightarrow y = \frac{1}{4}$$

$$\sin x = -1 \Rightarrow y = -\frac{5}{4}$$

$$\Rightarrow y = R_{\text{د}} \in [-\frac{5}{4}, \frac{1}{4}]$$

$$y = \frac{2 \sin x - 1}{\sin x + 3} \Rightarrow y \sin x + 3y = 2 \sin x - 1 \Rightarrow (y - 2) \sin x + 3y + 1 = 0$$

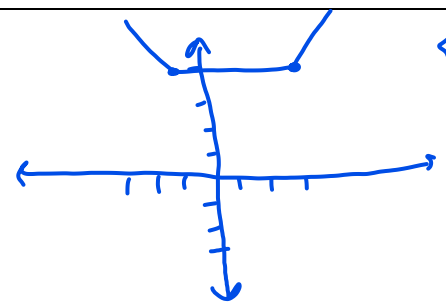
$$\Rightarrow \sin x = -\frac{3y + 1}{y - 2} \Rightarrow -1 \leq -\frac{3y + 1}{y - 2} \leq 1 \Rightarrow y \in [-\frac{3}{4}, \frac{1}{4}]$$

$$D_f = \{x \in \mathbb{R} \mid x^2 - x + 1 \neq 0\} \Rightarrow D_f = \mathbb{R} \quad \checkmark$$

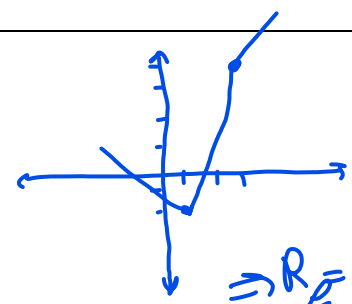
$$f(x) = \frac{x^2 - x + \frac{1}{x}}{x^2 - x + 1} = \frac{(x - \frac{1}{x})^2}{(x - \frac{1}{x})^2 + \frac{5}{x}} \quad \begin{matrix} (x - \frac{1}{x})^2 = 0 \Rightarrow f(x) = 0 \\ (x - \frac{1}{x})^2 = +\infty \Rightarrow f(x) = 1 \end{matrix} \quad \checkmark$$

$$\Rightarrow R_f \in [0, 1) \quad \checkmark$$

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$$\Rightarrow R_f = [0, +\infty) \quad \checkmark$$



$$\Rightarrow R_f = [-2, +\infty)$$

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$$|x-2| - |2x+2| \leq 1$$

$$x \geq 2: x-2-2x-2 \leq 1 \Rightarrow -x-4 \leq 1 \Rightarrow -5 \leq x \Rightarrow x \geq 2 \quad (1)$$

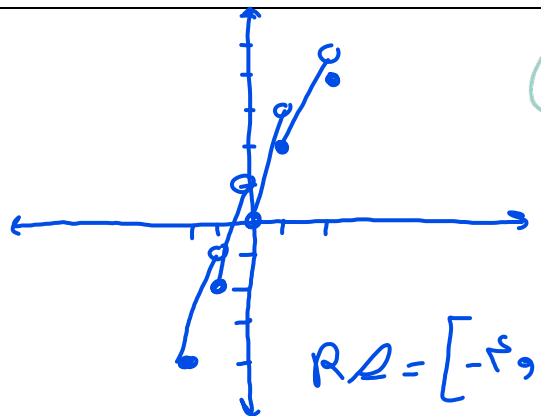
$$-1 \leq x \leq 2: 2-x-2x-2 \leq 1 \Rightarrow -3x \leq 1 \Rightarrow x \geq -\frac{1}{3} \Rightarrow -\frac{1}{3} \leq x \leq 2 \quad (2)$$

$$x < -1: 2-x+2x+2 \leq 1 \Rightarrow x+4 \leq 1 \Rightarrow x \leq -3 \Rightarrow x \leq -3 \quad (3)$$

$$x \in (-\infty, -3] \cup [-\frac{1}{3}, +\infty)$$

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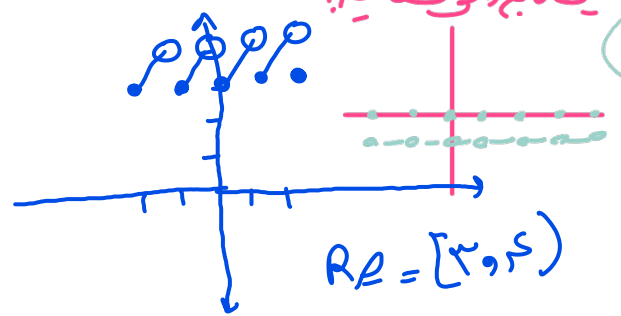
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$$R_f = [-\frac{4}{3}, \frac{2}{3})$$

$$y = 2 + [x] - \{x\}$$

این تابع را می توانیم به این شکل بنویسیم



$$R_f = [2, 3)$$

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$$y = \sin^2 x + \sin^2 x + 1$$

$$(1) \sin^2 x = 1 \rightarrow y = 3$$

$$(2) \sin^2 x = 0 \rightarrow y = 1$$

$$(3) \sin^2 x = -\frac{1}{2} \text{ (impossible)} \Rightarrow R_f \in [1, 3]$$

ب

$$y = \sin^2 x - \sin x + 1$$

$$(1) \sin x = 1 \rightarrow y = 1$$

$$(2) \sin x = -1 \rightarrow y = 3$$

$$(3) \sin x = \frac{1}{2} \rightarrow y = \frac{3}{4}$$

$$\Rightarrow R_f \in [\frac{3}{4}, 3]$$

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