

$$yx - x^2 - x - y = 0 \Rightarrow -x^2 + (y-1)x - y = 0 \Rightarrow \Delta \geq 0 \Rightarrow (y-1)^2 - 4(1+y) \geq 0$$

$$y^2 - 2y + 1 - 4 - 4y \geq 0 \Rightarrow y^2 - 6y - 3 \geq 0 \Rightarrow y \in (-\infty, -1] \cup [7, +\infty)$$

الف) $y \in \mathbb{R} / \text{ب) } \Delta = 0 \Rightarrow 3y - 2 = 0 \Rightarrow y = \frac{2}{3} \Rightarrow \Delta > 0 \Rightarrow y < \frac{2}{3} \Rightarrow y \in (-\infty, \frac{2}{3})$

$\Rightarrow \sqrt{y} = 2 \Rightarrow R \in [0, 2]$

ج) توان $\Rightarrow y \in \mathbb{R}$

د) $x^3 - 6x^2 + 11x + 6 = y' \Rightarrow y' \in \mathbb{R} \Rightarrow y = \log y' \Rightarrow y \in \mathbb{R}$

الف) $R - \{ \frac{1}{2} \} \Rightarrow R - \{ \frac{1}{2} \} = y$

ب) $y^2x - y^2 = x^2 + 1 \Rightarrow y^2x(x-1) = x^2 + 1 \Rightarrow$

$x = \frac{y^2+1}{y^2-1} \Rightarrow R - \{ \pm 1 \}$

ج) $\sqrt{x^2+3} + \frac{1}{\sqrt{x^2+3}} \Rightarrow \sqrt{x^2+3} > 0 \Rightarrow \sqrt{x^2+3} \geq \sqrt{3} \Rightarrow y \in [\sqrt{3}, +\infty)$

د) $x^2 + \frac{1}{x^2+1} = y \Rightarrow x^2 = t \Rightarrow t + \frac{1}{t+1} = y \Rightarrow t \rightarrow 0 \Rightarrow y = 1 \Rightarrow y \in [1, +\infty)$

الف) $y \Rightarrow yx^2 - y = 3x^2 + 1 \Rightarrow yx^2 - 3x^2 = y + 1 \Rightarrow \frac{y+1}{y-3} = x^2 \Rightarrow \pm \sqrt{\frac{y+1}{y-3}} = x$

$\Rightarrow y \in (-\infty, -1] \cup (3, +\infty)$

ب) $\Rightarrow x^2 \rightarrow 0 \Rightarrow y = -1 \Rightarrow y \in (-\infty, -1] \cup (3, +\infty)$

الف) $\Rightarrow y \sin \pi y - 2 \sin = 1 \Rightarrow \frac{1-y}{y-2} = \sin \alpha \Rightarrow -1 \leq \frac{1-y}{y-2} \leq 1 \Rightarrow \frac{1-y}{y-2} \leq 1$

$\frac{1-y}{y-2} \geq -1 \Rightarrow y \in [\frac{1}{2}, \frac{1}{2}]$

ب) $\Rightarrow \sin \pi y = 1 \Rightarrow y = \frac{1}{2}$

$\Rightarrow \sin \pi y = -1 \Rightarrow y = \frac{3}{2}$

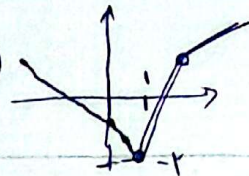
$\Rightarrow y \in [\frac{1}{2}, \frac{1}{2}]$

$$D_f \in \text{منوع } \Delta \Rightarrow \Delta < 0 \Rightarrow D_f \in \mathbb{R} / R_f \Rightarrow x^2 - x + \frac{1}{f} + \frac{3}{f} - \frac{3}{f}$$

$$\rightarrow 1 + \frac{-3}{f_2 x^2 - f_2 x + f} = y \Rightarrow y - 1 = \frac{-3}{f_2 x^2 - f_2 x + f} \Rightarrow \begin{cases} 2c = \frac{1}{f} = 8 \Rightarrow y - 1 = -1 \Rightarrow y = 0 \\ 2c \rightarrow \infty \Rightarrow y - 1 \rightarrow 0 \Rightarrow y \rightarrow 1 \end{cases}$$

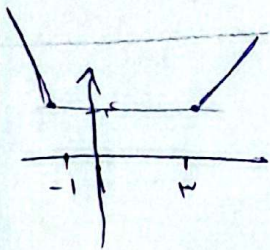
$$R_f \in [0, 1)$$

$$R_f \in [-1, +\infty)$$



$$\begin{aligned} x=0 &\Rightarrow y=-1 \\ x=1 &\Rightarrow y=-1 \\ x=2 &\Rightarrow y=1 \\ x=3 &\Rightarrow y=1 \end{aligned}$$

(الف)

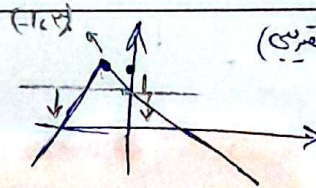


$$\Rightarrow R_f \in [1, +\infty)$$

(ب)

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$$\begin{aligned} -x + 2 - 2x - 2 \leq 1 &\Rightarrow -3x \leq 1 \\ x &\geq -\frac{1}{3} \quad (1) \end{aligned}$$



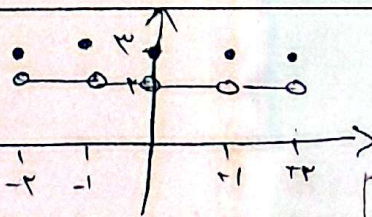
$$-1 - 2x - 2 \leq 1 \Rightarrow -3x \leq 1 \Rightarrow x \geq -\frac{1}{3}$$

$$-x + 2 + 2x + 2 \leq 1 \Rightarrow x \leq -3 \quad (2)$$

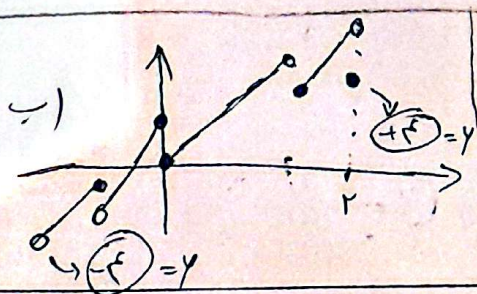
$$(1) \cup (2) \Rightarrow x \in (-\infty, -3] \cup [-\frac{1}{3}, +\infty)$$

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$$\text{الف) } [x] + [-x] + 3 \Rightarrow$$



$$R_f \in \{1, 3\}$$



$$y \in [-1, 1]$$

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$$\text{الف) } \frac{-b}{2a} \Rightarrow \sin x = \frac{1}{f} \Rightarrow \frac{1}{f} - \frac{1}{f} + 1 \Rightarrow y_{\min} = \frac{3}{f} / y_{\max} \Rightarrow \sin = -1$$

$$y \in [\frac{3}{f}, 3]$$

$$y_{\max} = 3$$

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$$\begin{aligned} \rightarrow \sin^2 t &\Rightarrow t^2 + t + 1 \rightarrow y_{\min} \Rightarrow t = \frac{1}{f} = \sin^2 \frac{1}{f} \Rightarrow y_{\min} = \frac{3}{f} \\ y &\in [\frac{3}{f}, 3] \rightarrow y_{\max} \Rightarrow t = 1 \Rightarrow y_{\min} = 3 \end{aligned}$$