

$$yx - x^2 - x - 2 = 0 \Rightarrow -x^2 + (y-1)x - 2 = 0 \Rightarrow \Delta \geq 0 \Rightarrow (y-1)^2 - 4 \geq 0$$

$$y^2 - 2y + 1 - 4 \geq 0 \Rightarrow y^2 - 2y - 3 \geq 0 \Rightarrow y \in (-\infty, -1] \cup [3, +\infty)$$

ext $\left| \frac{0}{\frac{1}{x}} \right| \xrightarrow{\Delta > 0} Rf_2 \left[-\frac{1}{x}, +\infty \right)$

الف) $y \in \mathbb{R} / \neg) \Delta = 0 \Rightarrow 3y - 2 = 0 \Rightarrow y = \frac{2}{3} \Rightarrow \Delta > 0 \Rightarrow \frac{-b}{2a} = x = \frac{2}{y-1} \Rightarrow y \neq 1$

$\Rightarrow y = 2 \Rightarrow R \in [0, 2]$

ب) توان $\Rightarrow y \in \mathbb{R}$

ایستادن $\Rightarrow y \in \mathbb{R}$

فراسط $\Rightarrow y \in \mathbb{R}$

۲) $x^3 - 6x^2 + 11x + 6 = y' \Rightarrow y' \in \mathbb{R} \Rightarrow y = \log y' \Rightarrow y \in \mathbb{R}$

الف) $R - \left\{ \frac{9}{2} \right\} \Rightarrow R - \left\{ \frac{3}{2} \right\} = y$

ب) $y^2 x - y^2 = x^2 + 1 \Rightarrow y^2 x^2 = y^2 + 1 \Rightarrow x = \frac{y^2 + 1}{y^2 - 1} \Rightarrow R - \left\{ \pm 1 \right\}$

۲) $\sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}} \Rightarrow \sqrt{x^2 + 3} > 0 \Rightarrow \sqrt{x^2 + 3} \xrightarrow{x \rightarrow \infty} \sqrt{3} \Rightarrow y \in [\sqrt{3}, +\infty)$

mind 2 $\sqrt{3} \rightarrow Rf_2 \left[\sqrt{3}, \frac{1}{\sqrt{3}} + \infty \right)$

۳) $x^2 + \frac{1}{x^2 + 1} = y \Rightarrow x^2 = t \Rightarrow t + \frac{1}{t+1} = y \xrightarrow{t \rightarrow \infty} y \rightarrow \infty \Rightarrow y \in \left[\frac{1}{2}, +\infty \right)$

الف) $y x^2 - y = 3x^2 + 2 \Rightarrow y x^2 - 3x^2 = y + 2 \Rightarrow \frac{y+2}{y-3} = x^2 \Rightarrow \pm \sqrt{\frac{y+2}{y-3}} = x$

$\Rightarrow y \in (-\infty, -2] \cup (3, +\infty)$

ب) $x^2 \rightarrow 0 \Rightarrow y = -2$

$x^2 \rightarrow \infty \Rightarrow y \rightarrow 3$

$\Rightarrow y \in (-\infty, -2] \cup (3, +\infty)$

الف) $y \sin x + 3y - 2 \sin x = 1 \Rightarrow \frac{1-3y}{y-2} = \sin x \Rightarrow -1 \leq \frac{1-3y}{y-2} \leq 1 \Rightarrow \frac{-1-3y}{y-2} \leq 1$

$\frac{-1-3y}{y-2} \geq -1$

الف) $\cap$ ب) $\Rightarrow y \in \left[\frac{3}{2}, \frac{1}{2} \right]$

ب) $\sin x = 1 \Rightarrow y = \frac{1}{2}$

$\sin x = -1 \Rightarrow y = \frac{3}{2}$

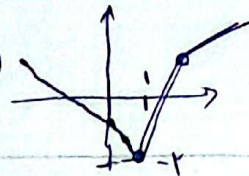
$\Rightarrow y \in \left[\frac{3}{2}, \frac{1}{2} \right]$

$$D_f \in \text{منوع } \Delta \Rightarrow \Delta < 0 \Rightarrow D_f \in \mathbb{R} / R_f \Rightarrow x^2 - x + \frac{1}{f} + \frac{3}{f} - \frac{3}{f}$$

$$\rightarrow 1 + \frac{-3}{f^2 x^2 - f^2 x + f} = y \Rightarrow y - 1 = \frac{-3}{f^2 x^2 - f^2 x + f} \Rightarrow \begin{cases} 2c = \frac{1}{f} = 8 \Rightarrow y - 1 = -1 \Rightarrow y = 0 \\ 2c \rightarrow \infty \Rightarrow y - 1 \rightarrow 0 \Rightarrow y \rightarrow 1 \end{cases}$$

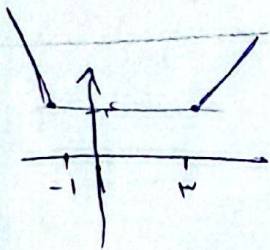
$$R_f \in [0, 1) \checkmark$$

$$R_f \in [-1, +\infty)$$



$$\begin{aligned} x=0 &\Rightarrow y=-1 \\ x=1 &\Rightarrow y=-1 \\ x=2 &\Rightarrow y=1 \\ x=3 &\Rightarrow y=0 \end{aligned}$$

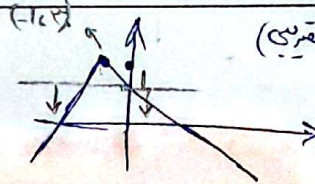
(الف)



$$\Rightarrow R_f \in [1, +\infty) \checkmark$$

7

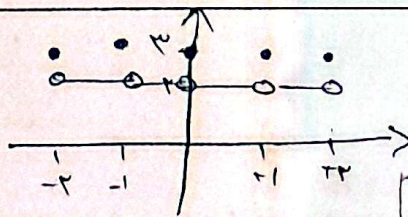
$$-x + 2 - 2x - 2 \leq 1 \Rightarrow -3x \leq 1 \Rightarrow x \geq -\frac{1}{3} \quad (1)$$



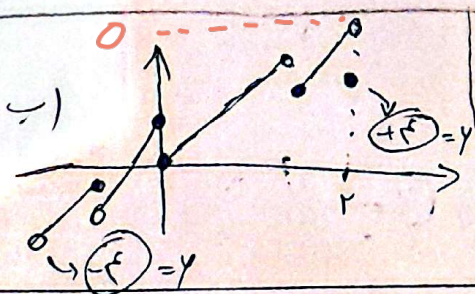
$$-1 - 2x - 2 \leq 1 \Rightarrow -3x \leq 1 \Rightarrow x \geq -\frac{1}{3} \quad (2)$$

$$-x + 2 + 2x + 2 \leq 1 \Rightarrow x \leq -3 \quad (3) \quad (1) \cup (2) \Rightarrow x \in (-\infty, -3] \cup [-\frac{1}{3}, +\infty)$$

$$\text{الف) } [x] + [-x] + 3 \Rightarrow$$



$$R_f \in \{2, 3\} \checkmark$$



$$y \in [-4, 4] \quad (-4, 0)$$

$$\text{الف) } \frac{-b}{2a} \Rightarrow \sin x = \frac{1}{f} \Rightarrow \frac{1}{f} - \frac{1}{f} + 1 \Rightarrow y_{\min} = \frac{3}{f} / y_{\max} \Rightarrow \sin = -1$$

$$y \in [\frac{3}{f}, 3]$$

$$y_{\max} = 3$$

$$\Rightarrow \sin^2 t = t \Rightarrow t^2 + t + 1 \rightarrow y_{\min} \Rightarrow t = \frac{1}{f} = \sin^2 \frac{1}{f} \Rightarrow y_{\min} = \frac{3}{f}$$

$$y_{\max} \Rightarrow t = 1 \Rightarrow y_{\min} = 3$$