مبین شهبازی

۱-

$y = x^3 - 3x^2 + 3x$

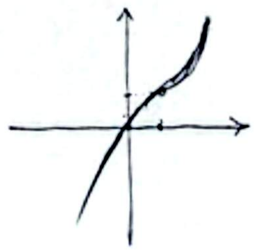
$y' = 3x^2 - 6x + 3$

$y' = 0 \qquad 3x^2 - 6x + 3 = 0 \qquad 3(x^2 - 2x + 1) = 0 \qquad 3(x-1)^2 = 0$

1

$+ \quad | \quad +$

نقاط بحرانی ندارد



۲-

$y = \dfrac{-x^3 + 4}{x^2} \qquad y' = \dfrac{-3x^2(x^2) - 2x(-x^3+4)}{x^4} = \dfrac{-3x^4 + 2x^4 - 8x}{x^4} = \dfrac{-x^4 - 8x}{x^4}$

$D_f = \mathbb{R} - \{0\}$

$-x^4 - 8x = 0 \qquad -x(x^3 + 8) = 0 \rightarrow x = 0$ غ ق ق

$x = \sqrt[3]{-8} = -2$ ق ق

۱ نقطه بحرانی

$y = \dfrac{x^3}{x^2 - 1} \qquad y' = \dfrac{3x^2(x^2-1) - 2x(x^3)}{(x^2-1)^2} = \dfrac{3x^4 - 3x^2 - 2x^4}{(x^2-1)^2} = \dfrac{x^4 - 3x^2}{(x^2-1)^2}$

$D_f = \mathbb{R} - \{\pm 1\}$

$x^4 - 3x^2 = 0 \qquad x^2(x^2 - 3) = 0 \rightarrow 0$ ق ق

$\pm\sqrt{3}$ ق ق

۳ نقطه بحرانی

$x^2 - 1 = 0 \qquad x = \pm 1$ غ ق ق

۳-

الف) $y = \dfrac{-x^2 + 4x + 1}{x - 1} \qquad y' = \dfrac{(-2x+4)(x-1) - (-x^2+4x+1)}{(x-1)^2} = \dfrac{-2x^2 + 6x - 4 + x^2 - 4x - 1}{(x-1)^2}$

$D_f = \mathbb{R} - \{1\}$

$\dfrac{-x^2 + 2x - 5}{(x-1)^2}$

نقطه بحرانی ندارد

$\Delta < 0$

ریشه ندارد

$-x^2 + 2x - 5 = 0 \qquad 4 - 4(-1)(-5) < 0$

$\boxed{x = 1 \text{ غ ق ق}}$

ب) $\dfrac{x^2 - 4x + 3}{x - 1} \qquad y' = \dfrac{(2x-4)(x-1) - (x^2 - 4x + 3)}{(x-1)^2} = \dfrac{2x^2 - 6x + 4 - x^2 + 4x - 3}{(x-1)^2}$

$y' = \dfrac{x^2 - 2x + 1}{(x-1)^2} \rightarrow \dfrac{\cancel{(x-1)^2}}{\cancel{(x-1)^2}} = 1$

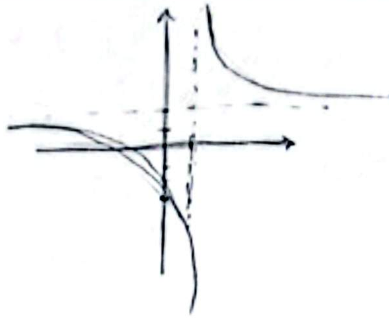
مشتق ثابت

اکسترمم ندارد

٤-

$y = \dfrac{2x+3}{x-1}$

$\lim\limits_{x\to\infty} \dfrac{2x+3}{x-1} = 2$



از ٤ ناحیه می‌گذرد

۵-

$y = \dfrac{ax+4}{x-b}$ $(2,3)$ $y = \dfrac{3x+4}{x-2}$ $\boxed{y^{-1} = \dfrac{2x+4}{x-3}}$

$x-b=0$ $\boxed{a=3}$

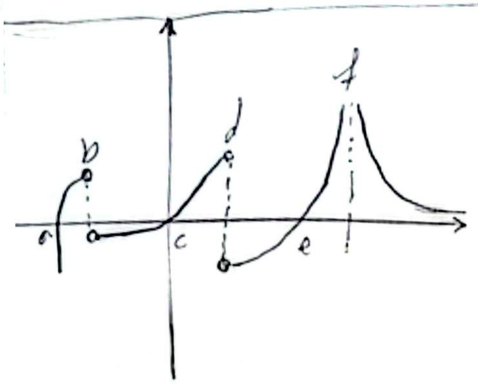
$x=b \rightarrow x=2 \rightarrow \boxed{b=2}$

۶-

$\dfrac{3x+1}{x-2}$ $y-3 = x-2$ $\lim\limits_{x\to\infty} \dfrac{3x+1}{x-2} = 3$ $\lim\limits_{x\to 2} \dfrac{3x+1}{x-2} = +\infty$

مرکز تقارن = $\begin{vmatrix} 2 \\ 3 \end{vmatrix}$ $\boxed{y = x+1 \text{ محور تقارن}}$

۷-



$\boxed{\text{۱ - ۶ نقطه}}$

-8

-9

$$y=\frac{x^2+2}{x^2+x+2}$$

$$y'=\frac{2x(x^2+x+2)-(2x+1)(x^2+2)}{(x^2+x+2)^2}=\cancel{2x^3}+2x^2+4x-\cancel{2x^3}+4x+x^2+2=\frac{3x^2+8x+2}{(x^2+x+2)^2}$$

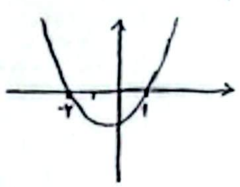
$$3x^2+8x+2=0 \qquad 64-4(3)(2)=64-24=40 \qquad x=\frac{-8\pm\sqrt{40}}{6}$$

$$\boxed{\frac{-8+\sqrt{40}}{6}\times\frac{-8-\sqrt{40}}{6}=\frac{64-40}{36}=\frac{24}{36}=\frac{4}{6}=\frac{2}{3}}$$

-10

$$y=x^2+ax+b$$

$$(x+2)(x-1)=x^2-x-2$$



$$\boxed{a=-1} \qquad \boxed{b=-2}$$

$$y_1=(x^2-x-2)^2 \qquad y_1'=2(x^2-x-2)(2x-1)$$

$$y_2=(x^2-x-2)^3 \qquad y_2'=3(x^2-x-2)^2(2x-1)$$

$$y_1'=0 \longrightarrow y_1'=2(x^2-x-2)(2x-1)$$

$$x=\tfrac{1}{2} \qquad x=-2,\ 1$$

$$y_1(\tfrac{1}{2})=(\tfrac{1}{4}-\tfrac{1}{2}-2)^2=\frac{81}{16}$$
$$-2.25$$

$$y_1(-2)=(4-(-2)-2)^2=16 \quad max$$
$$4+2$$

$$y_1(1)=(1-1-2)^2=4$$

$$y_2': \quad x=1,\ x=-2,\ x=\tfrac{1}{2}$$

$$y_2(1)=(1-1-2)^3=-8$$

$$y_2(-2)=(4+2-2)^3=64$$

y

	-2		1/2		1	
−		+		−		+
↘	64	↗	max	↘	-8	↗

$$\boxed{\begin{gathered}\left|\cancel{64-8}-\frac{81}{16}\right|=\frac{815}{16}\\ \left|56-\frac{81}{16}\right|\end{gathered}}$$

	-2		1/2		1	
−		+		−		+
↘	16	↗	81/16	↘	4	↗