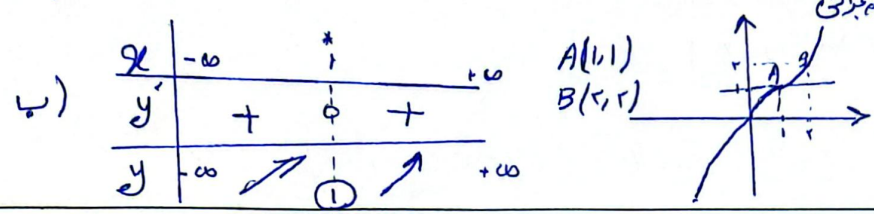


الف)  $y = x^3 - 3x^2 + 3x \Rightarrow D_f = \mathbb{R}$

الف)  $f'(x) = 0 \Rightarrow 3x^2 - 6x + 3 = 0 \Rightarrow 3(x-1)^2 = 0 \Rightarrow x = 1$  نقطه سرجانی



الف)  $y = \frac{-x^2 + 4}{x^2} \Rightarrow D_f = \mathbb{R} - \{0\} \Rightarrow f'(x) = 0 \Rightarrow \frac{-2x \cdot x^2 - (-x^2 + 4)(2x)}{(x^2)^2} = 0$

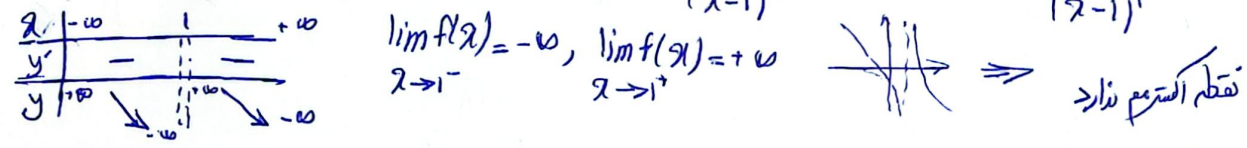
$\Rightarrow \frac{-2x^3 - 8x}{x^4} = \frac{-2(x^2 + 4)}{x^3} = 0 \Rightarrow \begin{cases} x=0 \text{ (ممنوع است)} \\ x=-2 \end{cases} \Rightarrow$  نقطه سرجانی دارد

ب)  $y = \frac{x^2}{x^2 - 1} \Rightarrow D_f = \mathbb{R} - \{\pm 1\}$  نقطه سرجانی دارد

$f'(x) = 0 \Rightarrow \frac{2x(x^2 - 1) - (x^2)(2x)}{(x^2 - 1)^2} = \frac{2x(x^2 - 2)}{(x^2 - 1)^2} \Rightarrow \begin{cases} x = -\sqrt{2} \\ x = \sqrt{2} \\ x = 0 \\ x = 1 \\ x = -1 \end{cases}$

الف)  $y = \frac{-x^2 + x + 1}{x - 1} \Rightarrow D_f = \mathbb{R} - \{1\}$   $\Delta < 0$

$f'(x) = \frac{(-2x + 1)(x - 1) - (-x^2 + x + 1)}{(x - 1)^2} = \frac{-2x^2 + 2x - 1 - (-x^2 + x + 1)}{(x - 1)^2} = \frac{-x^2 + x - 2}{(x - 1)^2}$



ب)  $y = \frac{x^2 - 4x + 3}{x - 1} \Rightarrow D_f = \mathbb{R} - \{1\} \Rightarrow y' = \frac{(2x - 4)(x - 1) - (x^2 - 4x + 3)}{(x - 1)^2} = \frac{2x^2 - 4x - x^2 + 4x - 3 + 4x - 3}{(x - 1)^2} = \frac{x^2 + 4x - 6}{(x - 1)^2} = 1 \neq 0$

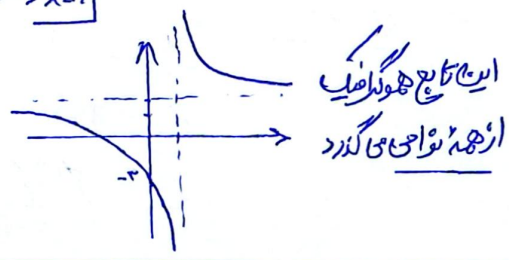
نقطه استرس ندارد

$y = x - 3$ ,  $D_f = \mathbb{R} - \{1\}$

$y = \frac{2x + 3}{x - 1}$  الف)  $\begin{cases} \text{مجاورت افقی: } y = \frac{a}{c} = 2 \\ \text{مجاورت عمودی: } x = -\frac{d}{c} = 1 \Rightarrow x - 1 = 0 \Rightarrow x = 1 \end{cases}$

ب)  $D_f = \mathbb{R} - \{1\}$  A(1, -3)

$R_f = \mathbb{R} - \{2\}$   $ad - bc < 0 \Rightarrow$  شاخه‌ها دوری



الف)  $w\left(\frac{-d}{c}, \frac{a}{c}\right) = w(r, r) \Rightarrow \frac{-d}{c} = r \Rightarrow \frac{-(-b)}{1} = r \Rightarrow b = r$   $y = \frac{ax + 4}{x - b}$

$\frac{a}{c} = r \Rightarrow \frac{a}{1} = r \Rightarrow a = r$

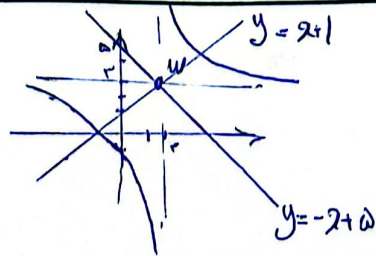
ب)  $y = \frac{3x + 4}{x - 2} \Rightarrow y(x - 2) = 3x + 4 \Rightarrow yx - 2y = 3x + 4 \Rightarrow yx - 3x = 2y + 4 \Rightarrow x = \frac{2y + 4}{y - 3}$

مقلوب  $y = \frac{3x + 4}{x - 2}$   
(x ≠ 2)

$$y = \frac{x^2 + 1}{x - 2}$$

$(y = \frac{a}{c} = 2)$  بجانب افق  
 $(2 = \frac{a}{c} = 2 \rightarrow a = 2c)$  بجانب قائم

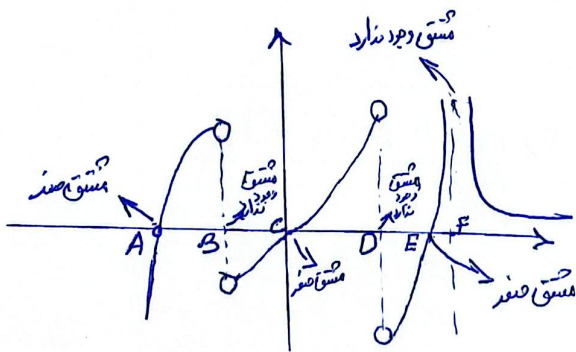
$$A|_{\frac{0}{2}}$$



$$W(2, 2)$$

$$y - y_w = \pm 1(x - x_w) \rightarrow y - 2 = \pm 1(x - 2)$$

$y = x + 1$   
 $y = -x + 5$



$$D_f = \mathbb{R}$$

در نقاط بحرانی نسبت تابع برابر ۰ یا  $\infty$  است

نقاطی که در آن‌ها  $f'$  یا مشتق صفر است: A, C, E

نقاطی که مشتق تعریف نشده است: B, D, F

نقطه بحرانی وجود دارد

$$y = |x^2 - ax + 2|, D_f = \mathbb{R} \rightarrow y' = \frac{(2x - a)(x^2 - ax + 2)}{|x^2 - ax + 2|} = 0 \rightarrow 2x - a = 0 \rightarrow x = \frac{a}{2}$$

$$x^2 - ax + 2 = 0 \rightarrow x = \frac{a \pm \sqrt{a^2 - 4}}{2}$$

در نقطه :  $x^2 - ax + 2 \xrightarrow{\Delta > 0} a^2 - 4 > 0 \rightarrow \begin{cases} a > 2 \\ a < -2 \end{cases}$

$$\rightarrow a^2 - 4 > 0 \rightarrow \begin{cases} a > 2 \\ a < -2 \end{cases}$$

در نقطه :  $y_{min} < 0 \xrightarrow{x = \frac{a}{2}} (\frac{a}{2})^2 - a(\frac{a}{2}) + 2 < 0 \rightarrow \begin{cases} a > 2 \\ a < -2 \end{cases}$

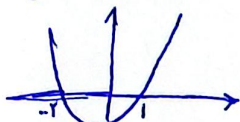
$$y = \frac{x^2 + 2}{x^2 + 2x + 2} \Rightarrow y' = \frac{2x(x^2 + 2x + 2) - (x^2 + 2)(2x + 2)}{(x^2 + 2x + 2)^2} = \frac{x^2 - 2}{(x^2 + 2x + 2)^2} = 0$$

$$\Rightarrow x = \pm \sqrt{2}$$

x	-√2	√2
y'	+	-
y	max	min

$$\Rightarrow \min \times \max = \frac{1}{1 - \sqrt{2}} \times \frac{1}{1 + \sqrt{2}} = \frac{1}{1 - 2} = -1$$

$$y = x^2 + ax + b$$



$$y = (x+1)/(x-1)$$

$$y = x^2 + 2x - 2$$

$$\rightarrow a = 1, b = -2$$

$$y = (x^2 + 2x - 2)^2$$

$$y' = 2(x^2 + 2x - 2)(2x + 2) = 0$$

$$\rightarrow x^2 + 2x - 2 = 0 \rightarrow x = 1, x = -2$$

$$2x + 2 = 0 \rightarrow x = -1$$

x	-2	-1	1
y'	-	+	-
y	min	max	min

$$\frac{11}{14}$$

$$y = (x^2 + 2x - 2)^2$$

$$y' = 2(x^2 + 2x - 2)(2x + 2) = 0$$

$$\rightarrow x^2 + 2x - 2 = 0 \rightarrow x = 1, x = -2$$

$$2x + 2 = 0 \rightarrow x = -1$$

x	-2	-1	1
y'	-	+	-
y	min	max	min

$$\frac{11}{14}$$

$$\text{max} - \text{min}$$

$$= (-\frac{1}{2}) - (-\frac{1}{2}) = 0$$