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نکته

تاریخچه

$$\lim_{x \rightarrow 0^+} \frac{f(x)}{x} \Rightarrow \lim_{x \rightarrow 0^+} f'(x) = \lim_{x \rightarrow 0^+} f'(x) = 4 \cos^2(x) \sin(x) + 4 \cos(x) = 4(1 - \frac{1}{2} \sin^2(x)) + 4 \cos(x)$$

$$\Rightarrow -12x + 4 \sin^2(x) + 4 \cos(x)$$

$$\lim_{x \rightarrow 0} \frac{f'(x)}{x} = \frac{-12x + 4 \sin^2(x) + 4 \cos(x)}{x} \Rightarrow 4 \sin^2(x) + 4 \cos(x) - 12 \Rightarrow a = 4$$

$$C \cdot \sin^2(x) + \cos(x) + b \Rightarrow 1 + b = 0 \Rightarrow b = -1$$

$$a + b = 4$$

$$f(x) = \sin^2(x) \Rightarrow f'(x) = 2 \sin(x) \cos(x) = \sin(2x) \Rightarrow \begin{matrix} \sin(2x) = a \\ -2x = \frac{1}{a} \end{matrix} \Rightarrow x = \pm \frac{1}{c}, a = \pm 1$$

$$\left. \begin{matrix} f(x) = f(\frac{1}{c}) = -\frac{1}{c} \\ f(x) = f(-\frac{1}{c}) = -\frac{1}{c} \end{matrix} \right\} \Rightarrow y_1 + y_2 = -\frac{2}{c}$$

$$f(x) = \frac{a}{x-1} \Rightarrow y = mx + b \Rightarrow m = \frac{4 - (-12)}{10 - (-10)} = \frac{16}{20} = \frac{4}{5} \Rightarrow y = \frac{4}{5}x + b \Rightarrow b = -9$$

$$L \rightarrow f'(x) = m \Rightarrow m = \frac{-a}{(x-1)^2} \Rightarrow 4 = \frac{-a}{(x-1)^2} \Rightarrow f(x) = y \Rightarrow \frac{a}{x-1} = 4x - 9 \Rightarrow a = (4x - 9)(x - 1)$$

$$\Rightarrow \Delta = 0 \Rightarrow 4x^2 - 14x + 9 = 0 \Rightarrow 4x^2 - 14x + 9 = 0$$

$$f(x) = \frac{-1}{x-1} \Rightarrow \frac{-1}{4} = \frac{1}{\mu} \Rightarrow \mu = -4 \Rightarrow 4x^2 - 14x + 9 = 0 \Rightarrow 4x^2 - 14x + 9 = 0$$

$$f(x) = \frac{a+9}{a(x+1)} = 1 \Rightarrow f(1) = \frac{1+a}{a+1} \Rightarrow h(x) = \frac{1+a}{a+1} \Rightarrow h(1) = \frac{1+a}{a+1} = 1$$

$$\Rightarrow 1 + b = 1 \Rightarrow b = 0 \Rightarrow f'(x) = \frac{(1-a)^2}{(a(x+1))^2} \Rightarrow 1 = \frac{1-a^2}{(1+a)^2} \Rightarrow 1 - a^2 = (1+a)^2$$

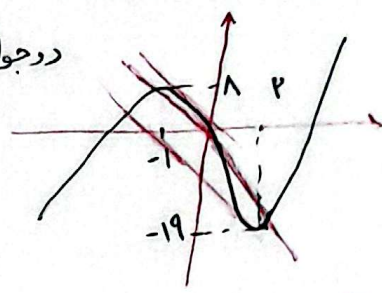
$$\Rightarrow 1 - a^2 + (1+a)^2 = 0 \Rightarrow (a+1)(1+a) = 0 \Rightarrow a = -1, b = 1 \Rightarrow a - b = -2$$

3) $f(x) = \sin x + \frac{1}{c} \cos x, g(x) = \frac{1}{c} \sin x \Rightarrow \sin x + \frac{1}{c} \cos x - \frac{1}{c} \sin x$

$\Rightarrow \frac{1}{c} \cos x + \frac{1}{c} \sin x \Rightarrow x = \frac{\pi}{4}, f'(x) = \cos x - \frac{1}{c} \sin x \Rightarrow f'(\frac{\pi}{4}) = \frac{\sqrt{c}}{c} - \frac{\sqrt{c}}{c}$
 $\Rightarrow \frac{\sqrt{c}}{c} \Rightarrow f(\frac{\pi}{4}) = \frac{\sqrt{c}}{c} \Rightarrow y - \frac{\sqrt{c}}{c} = \frac{\sqrt{c}}{c} (x - \frac{\pi}{4}) \Rightarrow y = \frac{\sqrt{c}}{c} x - \frac{\sqrt{c}}{c} \frac{\pi}{4} + \frac{\sqrt{c}}{c} = \frac{\sqrt{c}}{c} x - \frac{\sqrt{c}}{c} \frac{\pi - 4}{4}$

4) $f(x) = 2x^3 - 3x^2 - 12x + 1 \Rightarrow f'(x) = 6x^2 - 6x - 12 \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0$

$y = -9x + 1 \Rightarrow 4x^3 - 4x - 12x + 1 = -9x + 1 \Rightarrow 4x^3 - 4x + 13 = 0 \Rightarrow \Delta > 0$ در جواب دارد



5) $f(x) = Kx^3 + (K+1)x^2 \Rightarrow f'(x) = 3Kx^2 + 2(K+1)x \Rightarrow f''(x) = 6Kx + 2(K+1)$

$\Rightarrow 4Kx + 2K + 2 = 0 \Rightarrow x = -\frac{(K+1)}{2K} < 0 \Rightarrow \begin{cases} K > 0 \\ K < -1 \end{cases}$

$f''(x) > 0 \Rightarrow K \left(\frac{-(K+1)}{2K} \right)^2 + (K+1) \left(\frac{-(K+1)}{2K} \right) > 0 \Rightarrow \frac{-(K+1)^2}{4K} + \frac{-(K+1)^2}{2K} > 0 \Rightarrow \frac{2(K+1)^2}{4K} > 0$

$\Rightarrow (K+1)^2 > 0 \Rightarrow K+1 > 0 \Rightarrow K > -1$ و 5) $\rightarrow K > 0$ بازای هیچ حدی

6) $y = x^3 + ax^2 + bx - 1 \Rightarrow y' = 3x^2 + 2ax + b \Rightarrow y'' = 6x + 2a \Rightarrow f''(-1) = 0 \Rightarrow -4 + 2a = 0$

$\Rightarrow a = 2, f(-1) = -1 \Rightarrow -1 + 2b - 1 = -1 \Rightarrow b = \frac{1}{2} \Rightarrow \frac{a}{b} = \frac{4}{1}$

7) $f(x) = ax^3 + ax^2 + bx - 1 \Rightarrow f'(x) = 3ax^2 + 2ax + b \Rightarrow f'(0) = 0 \Rightarrow b = 0, f(0) = -1 \Rightarrow C = \frac{1}{9}$

$f(x) = a(x^3 + x^2) \Rightarrow \begin{cases} a > 0 \\ a < -\frac{2a}{3} \end{cases} \Rightarrow \min f(x) = f(-\frac{2a}{3}) \Rightarrow \frac{-1a}{27} + \frac{4a}{9} + f = 0 \Rightarrow \frac{3a}{27} = -f \Rightarrow a^3 = -27 \Rightarrow a = -3 \Rightarrow \min f(x) = \frac{2}{3}$

8) $f(x) = x^3 - 4x^2 + 4x \Rightarrow f'(x) = 3x^2 - 8x + 4 \Rightarrow \Delta = 64 - 48 = 16 \Rightarrow x = 1, 2$

x	$-\infty$	1	2	$+\infty$
f'	$-$	$+$	$-$	$+$
f	\searrow	\nearrow	\searrow	\nearrow

$\Rightarrow \min f(x) = -\frac{16}{27}$ و $\frac{16}{27}$ م. ا. ب. $\Rightarrow f'(a) = 3a^2 - 8a + 4 = 0 \Rightarrow a = 1 \Rightarrow C = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$