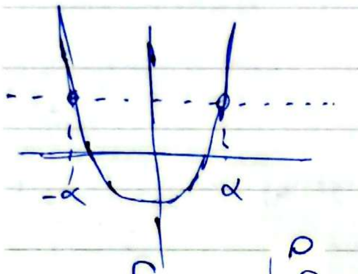
۱- $f'(x) = 3 \times 2 \times \cos^2(2x) \times (-\sin(2x)) + 2ax \Rightarrow \text{if } x = 0 \Rightarrow f'(0) = 0$, $b = 0 \Leftarrow \lim_{x\to 0} \frac{f(x) - f(0)}{x - 0} = 0$

$\lim_{x\to 0} \frac{f'(x) - f'(0)}{x - 0} = 2 \Rightarrow f''(0) = 2$

$\Rightarrow f''(0) = 2a \Rightarrow a = 1 \Rightarrow a + b = 1$



$f(x) = x^2 - 1 \Rightarrow f'(x) = 2x$, $\Rightarrow f'(\alpha) \times f'(-\alpha) = -1$

$\Rightarrow -4\alpha^2 = -1 \Rightarrow \alpha^2 = \frac{1}{4} \Rightarrow \alpha = \pm\frac{1}{2}$

$\Rightarrow f(x) \Rightarrow \text{if } x = \pm\frac{1}{2} = \frac{1}{4} - 1 = \frac{3}{4} \times 2 = -\frac{3}{2}$

۳- $y - 4 = 4(x - 2/5) \Leftarrow 4 = m = \frac{4 - (-2)}{2/5 - (-0/5)} \Leftarrow$ [illegible]

$y = 4x - 9 \Rightarrow 4x - 9 = \frac{a}{3x - 1} \longrightarrow 12x^2 - 24x + 9 - a = 0 \Rightarrow \Delta = 0$

$\Rightarrow (-24)^2 - 4(12)(9 - a) = 0 \Rightarrow 48a = -144 \Rightarrow a = -3 \Rightarrow f(5)$

$= \frac{-3}{9} = -\frac{1}{3}$

۴- $\frac{1 - a^2}{(a+1)^2} = 2 \Leftarrow 2 = \frac{1 \times (ax+1) - a(x+a)}{(ax+1)^2}$, $\text{if } x = 1 \Rightarrow 2x + b = \frac{x + a}{ax + 1}$

$\Rightarrow 2a^2 + 4a + 2 - 1 + a^2 = 0 \Rightarrow (a+1)(3a+1) = 0$ $a = -1$ یا $a = -\frac{1}{3}$

$\Rightarrow y = \frac{x - \frac{1}{3}}{-\frac{1}{3}x + 1} = \frac{3x - 1}{-3 + x}$ تابع ثابت می‌شود ⇐ غ ق ق

$\xrightarrow{\text{if } x=1} y = 1$

$\Rightarrow 2 + b = 1 \Rightarrow b = -1$ ⇐ نقطه (۱,۱) روی خط

$\Rightarrow a - b = -\frac{1}{3} + 1 = \frac{2}{3}$

$f(x)=g(x) \Rightarrow \sin x + \frac{1}{2}\cos x = \frac{3}{2}\sin x \Rightarrow \frac{1}{2}\sin x = \frac{1}{2}\cos x \Rightarrow \sin x = \cos x$ —۵

$\Rightarrow x = \frac{\pi}{4} \Rightarrow \left(\frac{\pi}{4}, \frac{3\sqrt{2}}{4}\right)$, $f'(x) = \cos x - \frac{1}{2}\sin x \Rightarrow$ if $x = \frac{\pi}{4}$

$f'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{4} \Rightarrow d: y - \frac{3\sqrt{2}}{4} = \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)$

$\Rightarrow y=0 \Rightarrow \frac{-3\sqrt{2}}{4} = \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right) \Rightarrow x = \frac{\pi}{4} - 3$

$f'(x) = 6x^2 - 6x - 12 = 0 \Rightarrow x^2 - x - 2 = 0 \Rightarrow x=-1,\ x=2$, $m_{AB} = \frac{-19-8}{2-(-1)} = -9$ —۶

$\Rightarrow 6x^2 - 6x - 12 = -9 \Rightarrow 6x^2 - 6x - 3 = 0 \xrightarrow{\Delta > 0}$ ۲ نقطه

$y = kx^3 + (k+1)x^2 \Rightarrow y' = 3kx^2 + 2(k+1)x \Rightarrow y'' = 6kx + 2k + 2 = 0$ —۷

$k\left(\frac{-(k+1)}{3k}\right)^3 + (k+1)\left(\frac{-(k+1)}{3k}\right)^2 > 0 \Rightarrow \frac{-(k+1)^3 + 3(k+1)^3}{27k^2} > 0$

$\Rightarrow \frac{2(k+1)^3}{27k^2} > 0$ $\quad -1 \quad 0$: $- \quad + \quad +$

$6kx = -2k-2 \Rightarrow x = \frac{-(k+1)}{3k} < 0 \Rightarrow \frac{k+1}{3k} > 0 \Rightarrow$ $\quad -1 \quad 0$: $+ \quad - \quad +$

$\Rightarrow k \in (0, +\infty) \Rightarrow$ جهت تقعر رو به بالاست

$y = x^3 + ax^2 + bx - 1 \Rightarrow y' = 3x^2 + 2ax + b \Rightarrow y'' = 6x + 2a$ —۸

$y''=0 \Rightarrow 6x + 2a = 0 \Rightarrow 3x + a = 0 \Rightarrow x = -\frac{a}{3}$ $\quad -\frac{a}{3} = -1 \Rightarrow a = 3$

$y = x^3 + 3x^2 + bx - 1 \Rightarrow$ if $x=-1 \Rightarrow -1 + 3 - b - 1 = -4 \Rightarrow b = 5$

$\frac{a}{b} = \frac{3}{5} = 0.6$

$f(x) = x^3 + ax^2 + bx + c \to f(0) = 4 \to c = 4 \to f'(x) = 3x^2 + 2ax + b$ ۹-

$\to f'(0) = 0 \to b = 0 \to f(x) = x^3 + ax^2 + 4 \to f'(x) = 3x^2 + 2ax$

$\to x(3x + 2a) = 0,\ x = 0,\ \left(x = -\frac{2a}{3}\right) \to$ مینیمم

$f\left(-\frac{2a}{3}\right) = 0 \to \left(-\frac{2a}{3}\right)^3 + a\left(-\frac{2a}{3}\right)^2 + 4 = 0 \to a = -3 \to -\frac{2a}{3}$

$= \frac{-2(-3)}{3} = 2$

$f(x) = x^4 - 6x^2 + 5 \to f'(x) = 4x^3 - 12x = 0 \to x^3 - 3x = 0$ ۱۰-

$x(x^2 - 3) = 0 \to$

x		$-\sqrt{3}$		0		$\sqrt{3}$	
$f(x)$	$-$		$+$		$-$		$+$
$f'(x)$	$\searrow$	-4	$\nearrow$		$\searrow$	-4	$\nearrow$

$\to f''(x) = 12x^2 - 12 = 0$

$x^2 - 1 = 0 \to x = \pm 1$

$A \begin{vmatrix} -\sqrt{3} \\ -4 \end{vmatrix}$ $B \begin{vmatrix} \sqrt{3} \\ -4 \end{vmatrix}$

$C \begin{vmatrix} 1 \\ 0 \end{vmatrix}$ $D \begin{vmatrix} -1 \\ 0 \end{vmatrix}$

$m_{AB} = 0$

$m_{CD} = 0$

موفق باشید عطاریان