۱- حسین شهبازی ۱۵

$f(x)=\cos^3(2x)+ax^2+b$

$\lim_{x\to 0}\frac{f(x)}{x}=\frac{\cos^3 x+ax^2+b}{x}\xrightarrow{\text{هم‌ارزی}}\frac{(2x)^3+ax^2+b}{x}=0\xrightarrow{\text{مشتق‌پذیر}} b=0$

۰/۵

$\lim_{x\to 0}\frac{f'(x)}{x}=2\longrightarrow\frac{6\cos^2(2x)\times-\sin 2x+2ax}{x}\xrightarrow{\text{هم‌ارزی}}\frac{6(2x)^2\times-(2x)+2ax}{x}=\frac{-6(2x)^3+2ax}{x}$

$f'(x)=3\cos^2(2x)\times-\sin 2x\times 2=6\cos^2(2x)\times-\sin 2x+2ax$

جواب صفحه بعد

$a+b=1+(0)=1$

$\frac{-6\times 8x^3+2ax}{x}=\frac{\cancel{x}(-48x^2+2a)}{\cancel{x}}=2 \qquad 2a=2 \qquad \boxed{a=1}$

۲-

$f(x)=x^2-1$

$f'(x)=2x$

$m_1=-\frac{1}{m_2} \qquad \frac{y_1-y_0}{x_1-0}=-\frac{1}{\frac{y_2-y_0}{x_2-0}} \qquad x\to\{\frac{1}{2},-\frac{1}{2}\}$

$x_1=-x_2 \qquad 2x_1=-\frac{1}{2x_2} \qquad 4x_1x_2=-1$

$-4x_1^2=-1 \qquad \boxed{x_1=\frac{1}{2},\ x_2=-\frac{1}{2}}$

$f(\frac{1}{2})=(\frac{1}{2})^2-1=-\frac{3}{4}$

$f(-\frac{1}{2})=(-\frac{1}{2})^2-1=-\frac{3}{4}$

$\boxed{f(\frac{1}{2})+f(-\frac{1}{2})=-\frac{3}{2}}$

۲

۳-

$m_d=\frac{\Delta y}{\Delta x}\longrightarrow\frac{(6-(-12))}{2.5-(-0.5)}=\frac{18}{3}=6$

$d=6x-9$

$f(x)=\frac{a}{2x-1}$

$\frac{a}{2x-1}=6x-9 \qquad a=3(2x-3)(2x-1)\Rightarrow 3(4x^2-2x-6x+3)=a$

$12x^2-24x+9-a=0$, $\Delta=0$ $\longrightarrow (24)^2-4(12)(9-a)=0$

$4((144)-(12)(9-a))$

$9-a=12$

$\boxed{a=-3}$

$\boxed{f(x)=\frac{-3}{2x-1}}$

$\boxed{f(5)=\frac{-3}{10-1}=\frac{-3}{9}=-\frac{1}{3}}$

۲

$\frac{x+a}{ax+1}$

سوال ۴

مشتق تابع $f'(1)=g'(1) \to \frac{1-a^2}{(a+1)^2}=2 \to \frac{(1-a)(1+a)}{(1+a)^2}=2 \to 2a+2=1-a \to \boxed{a=-\frac{1}{3}}$

مقدار تابع $f(1)=g(1) \to \frac{1-\frac{1}{3}}{-\frac{1}{3}+1}=2+b \to \boxed{b=-1}$

$a-b=\frac{2}{3}$

۰

۵-

$\sin x+\frac{1}{2}\cos x=\frac{3}{2}-\sin x$

$\frac{1}{2}\cos x=\frac{1}{2}\sin x \qquad \cos x=\sin x \qquad x=k\pi+\frac{\pi}{4} \qquad [0,\pi] \to \boxed{x=\frac{\pi}{4}}$

$f(\frac{\pi}{4})=\frac{3\sqrt{2}}{4} \qquad f(\frac{\pi}{4})=y \qquad y=mx+b \to \frac{3\sqrt{2}}{4}=\frac{\sqrt{2}}{4}\underset{\frac{\pi}{4}}{\underset{\downarrow}{x}}+b \qquad b=\frac{3\sqrt{2}}{4}-\frac{\sqrt{2}\pi}{16}$

$f'(x)=\sin x-\frac{1}{4}\cos x$

$f'(\frac{\pi}{4})=m \to \boxed{\frac{\sqrt{2}}{4}=m}$

$0=\frac{\sqrt{2}}{4}x+\left(\frac{3\sqrt{2}}{4}-\frac{\sqrt{2}\pi}{16}\right) \to \frac{4\sqrt{2}x+12\sqrt{2}-\sqrt{2}\pi}{16}=0$

$\sqrt{2}\left(\frac{4x+12-\pi}{16}\right)=0$

$4x+12-\pi=0$

$\boxed{x=\frac{\pi-12}{4}}$

۴

۶-

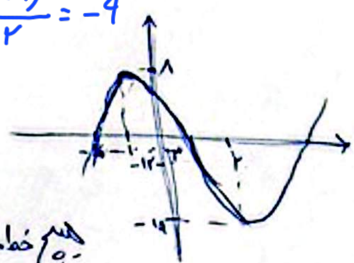
$f(x)=2x^3-3x^2-12x+1$

$f'(x)=6x^2-6x-12 \qquad 6(x^2-x-2) \to 6(x-2)(x+1)=0$

x		-1		2	
y'	+	0	−	0	+
y	↗	8	↘	-19	↗

$m_1=\frac{\Delta y}{\Delta x} \to \frac{8-(-19)}{-1-(2)}=\frac{27}{-3}=-9$

$d=-9x-1 \qquad A\begin{vmatrix}-1\\8\end{vmatrix} \quad B\begin{vmatrix}2\\-19\end{vmatrix} \to m_{AB}=\frac{8-(-19)}{-1-2}=-9$



عرض $=\frac{y_{max}+y_{min}}{2}=\frac{8+(-19)}{2}=-\frac{11}{2}$

طول $=\frac{-b}{3a} \to \frac{-(-3)}{6}=\frac{1}{2}$

هیچ خط مماس دیگری با شیب -9 در منحنی وجود ندارد چون خط از نقطه عطف منحنی عبور کرده است

$f'(x)=6x^2-6x-12=-9 \to 6x^2-6x-3=0$

$\Delta>0 \to$ دو نقطه یافت می‌شود

۱

۷-

$y = kx^3 + (k+1)x^2$ $x<0$

$-\frac{k+1}{3k}K + k+1 > 0 \to -\frac{k+1}{3} + k+1 > 0 \to \frac{2k+2}{3} > 0 \to k+1>0 \to \boxed{k>-1 \ (II)}$

$y' = 3kx^2 + 2(k+1)x$ $\boxed{k<-1 \,,\, k>0 \ (I)}$

$y'' = 6kx + 2(k+1)$ $(I) \cap (II) \to k>0 \to$ هیچ مقدار صحیح و منفی نیست

عطف: $y''=0$ $6kx + 2(k+1) = 0$ $6kx = -2(k+1)$ $x = \frac{-2(k+1)}{6k} \to -\frac{(k+1)}{3k} < 0$

k		-1		0	
y''	$-$		$+$		$-$

$\{k \in \mathbb{Z} , k<0 \mid k \in (-\infty, -1]\}$

۸-

$f(x) = x^3 + ax^2 + bx + 1$ $-1 + a - b + 1 = -4$

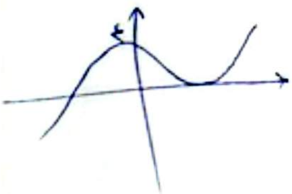
$(-1, -4)$ $a - b = -2$

عطف $x = \frac{-b}{3a} \to x = \frac{-a}{3} \to \frac{-a}{3} = -1 \to \boxed{a=3}$ $\frac{a}{b} = \frac{3}{5}$

$-4 = -1 + 3 - b - 1 \to \boxed{b=5}$

۹

$f(x) = x^3 + ax^2 + bx + c$ $f(0) = 4 \to c = 4$ $f(x) = x^3 + ax^2 + 4 \to f(x) = x^3 - 3x^2 + 4$



$f'(0) = 0$ $3x^2 + 2ax + b = 0 \to b = 0$ $f'(x) = 3x^2 - 6x = 0$

$3x(x-2) = 0 \ \boxed{x \to 2}$

$f''(x) = 6x + 2a = 0 \to x = -a/3$

$f(-a/3) = \frac{-a^3}{27} + \frac{a^3}{9} + 4 = 2$ $\frac{2a^3}{27} = -2$ $\frac{a^3}{27} = -1 \ \boxed{a = -3}$

$\frac{y_{max} + y_{min}}{2} = y$ عطف $\frac{4+0}{2} = 2$

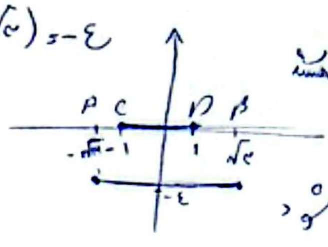
۱۰-

$f(x) = x^4 - 6x^2 + 5$ $f(0) = 5$

$f(\sqrt{3}) = -4$ $f(-\sqrt{3}) = -4$

$f'(x) = 4x^3 - 12x$ $4x(x^2 - 3) = 0$ $x = 0$, $x = \pm\sqrt{3}$

$f''(x) = 12x^2 - 12$ $12(x^2 - 1) = 0$ $x = \pm 1$ $f(1) = 0$, $f(-1) = 0$



۲ خط موازی با هم هستند
زاویه‌ای بین
آنها تشکیل نمی‌شود

← زاویه صفر درجه

سؤال ۱۱

$f(x)=\cos^3(2x)+ax^2+b$

$\lim_{x\to 0^+}\frac{f(x)}{x}=0 \longrightarrow \lim_{x\to 0^+}\frac{\cos^3(2x)+ax^2+b}{x}=0 \longrightarrow \lim_{x\to 0^+}\frac{1+b}{x}=0 \longrightarrow \boxed{b=-1}$

$\lim_{x\to 0^-}\frac{f'(x)}{x}=2 \longrightarrow \lim_{x\to 0^-}\frac{-6\sin(2x)\cos^2(2x)+2ax}{x}=2 \xrightarrow{\text{هم ارزی}} \lim_{x\to 0^-}\frac{-6\times 2x+2ax}{x}=2$

$\longrightarrow \lim_{x\to 0^-}\frac{(2a-12)x}{x}=2 \longrightarrow 2a-12=2 \longrightarrow \boxed{a=7}$

$a+b=6$