نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ...۲۷... کلاس .....

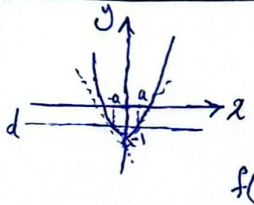
$f(x)=\cos^3(\pi x)+ax^2+b \quad \lim_{x\to 1}\frac{f(x)}{x}=0 \xrightarrow{x\to 1} \cos^3(\pi x)=1 \Rightarrow f(x)=1+b$

$\Rightarrow \lim_{x\to 1}\frac{f(x)}{x}=\lim_{x\to 1}\frac{1+b}{x}=0 \Rightarrow 1+b=0 \Rightarrow b=-1 \Rightarrow f(x)=\cos^3(\pi x)+ax^2-1$

$f'(x)=-4\cos^2(\pi x)\sin\pi x+2ax \xrightarrow{x\to 0} f'(0)=-4(1)^2(0)+2a(0)=0$

$\lim_{x\to 0}\frac{f'(x)}{x}=\lim_{x\to 0}\frac{-4\cos^2(\pi x)\sin\pi x+2ax}{x}=1 \Rightarrow -4\cos^2(0)\sin 0+2a=2a \Rightarrow a=1$

$\Rightarrow a+b=0$



$f'(a)\cdot f'(-a)=-1$ (عمودند) $\Rightarrow 2a\times(-2a)=-1 \Rightarrow a^2=\frac{1}{4}$

$f(x)=x^2-1 \Rightarrow f'(x)=2x$

$f(a)=f(-a)=a^2-1=\frac{1}{4}-1=\frac{-3}{4}$

$A\left|\frac{1}{2},-\frac{3}{4}\right. \quad B\left|-\frac{1}{2},-\frac{3}{4}\right.$

مجموع عرض های این دو نقطه $=\frac{-3}{4}+\left(\frac{-3}{4}\right)=\frac{-3}{2}$

$A(-2,-12), B(3,4) \Rightarrow m_{AB}=\frac{4-(-12)}{3-(-2)}=\frac{16}{5}=4 \Rightarrow y-4=4(x-3) \Rightarrow y=4x-8$

$f'(x)=\frac{-2a}{(2x-1)^2}=4, \quad f(x)=\frac{a}{2x-1}=4x-8 \Rightarrow \begin{cases} a=-2(2x-1)^2 \\ a=(2x-1)(4x-8)\end{cases}$

$\Rightarrow -2(2x-1)^2=(2x-1)(4x-8) \Rightarrow -4x+2=4x-8 \Rightarrow x=1 \Rightarrow a=-2(2(1)-1)^2=-2$

$f(0)=\frac{-3}{10-1}=\frac{-3}{9}=\frac{-1}{3}$

$y'=\frac{1-ax^2}{(ax+1)^2} \Rightarrow y'(1)=\frac{1-a^2}{(a+1)^2}=2 \Rightarrow 2(a^2+2a+1)=1-a^2 \Rightarrow 3a^2+4a+1=0 \Rightarrow (a+1)(3a+1)=0$

$y=2x+b \Rightarrow y'=2$

$\Rightarrow \begin{cases} a=-1 \times \\ a=-\frac{1}{3} \checkmark \end{cases}$ دیگر هذلولی نیست و دایره است

$\Rightarrow y=\frac{x-\frac{1}{3}}{-\frac{1}{3}x+1}=\frac{3x-1}{-x+3} \xrightarrow{x=1} y=\frac{3-1}{-1+3}=1$

$y=2x+b \xrightarrow{(1,1)} 1=2+b \Rightarrow b=-1$

$\Rightarrow a-b=-\frac{1}{3}-(-1)=\frac{2}{3}$

$\sin x+\frac{1}{2}\cos x=\frac{\sqrt{2}}{2}\sin x \Rightarrow \sin x=\cos x \Rightarrow x=k\pi+\frac{\pi}{4} \xrightarrow{[0,\pi]} x=\frac{\pi}{4}$

$f\left(\frac{\pi}{4}\right)=\sin\frac{\pi}{4}+\frac{1}{2}\cos\frac{\pi}{4}=\frac{\sqrt{2}}{4}+\frac{\sqrt{2}}{4}=\frac{\sqrt{2}}{2}$

$f'(x)=\cos x-\frac{1}{2}\sin x \Rightarrow f'\left(\frac{\pi}{4}\right)=\cos\frac{\pi}{4}-\frac{1}{2}\sin\frac{\pi}{4}=\frac{\sqrt{2}}{4}-\frac{\sqrt{2}}{8}=\frac{\sqrt{2}}{8}$

$y-\frac{\sqrt{2}}{4}=\frac{\sqrt{2}}{8}\left(x-\frac{\pi}{4}\right) \Rightarrow y=\frac{\sqrt{2}}{8}x-\frac{\sqrt{2}\pi}{32}+\frac{\sqrt{2}}{4} \xrightarrow{y=0} x=\frac{\frac{\sqrt{2}\pi}{32}-\frac{\sqrt{2}}{4}}{\frac{\sqrt{2}}{8}}=\frac{\pi-8}{4}=\frac{\pi}{4}-2$

$f'(x) = 4x^3 - 4x - 12 = 0 \rightarrow x^3 - x - 3 = 0 \rightarrow x = -1, x = 3 \rightarrow \begin{cases} A(-1,1) \\ B(3,-19) \end{cases}$

$\rightarrow m_{AB} = \frac{1-(-19)}{-1-3} = -5$

$\rightarrow 4x^3 - 4x - 12 = -5 \rightarrow 4x^3 - 4x - 7 = 0 \xrightarrow{\Delta > 0}$ ۲ نقطه

$y = kx^3 + (k+1)x^2$

$x = \frac{-b}{3a} = -\frac{(k+1)}{3k} < 0$ نقطه مطلوب

$\rightarrow \frac{-1}{3} \quad 0$ (+ در k، − در 1)

$\rightarrow k < -1$ I

$y = k\left(-\frac{(k+1)}{3k}\right)^3 + (k+1)\left(\frac{-(k+1)}{3k}\right)^2 > 0$ تابع

$\rightarrow \frac{(k+1)^3}{(3k)^2}\left(\frac{1}{3}\right) > 0$ ($+$)

$k < 0 \Rightarrow$ $-1 \quad 0$ ($-$ | $+$ | $+$) $\rightarrow k = (-1, 0)$ II $\Rightarrow$ I ∩ II = ∅

به ازای هیچ مقدار صحیح و منفی k

$y = x^3 + ax^2 + bx - 1 \rightarrow y' = 3x^2 + 2ax \rightarrow y'' = 6x + 2a = 0 \rightarrow x = \frac{-2a}{6} = \frac{-a}{3}$ | $\frac{1}{2}$ نقطه عطف $x = \frac{-b}{3a} = \frac{-a}{3}$

$\rightarrow \frac{-a}{3} = -1 \rightarrow a = 3$ | $\rightarrow y = x^3 + 3x^2 + bx - 1 \xrightarrow{(-1,-4)} -4 = -1 + 3 - b - 1 \rightarrow b = 3$ |

$\rightarrow \frac{a}{b} = \frac{3}{3} = 1$

A | x $\rightarrow f(0) = 4 \rightarrow c = 4$ | $\rightarrow f(x) = x^3 + ax^2 + bx + 4$ ، $f'(x) = 3x^2 + 2ax + b$

نسبی max

$\rightarrow f'(0) = 0 \rightarrow b = 0 \rightarrow f(x) = x^3 + ax^2 + 4 \rightarrow f'(x) = 3x^2 + 2ax = x(3x + 2a)$

$f'(x) = 0 \rightarrow \begin{cases} x = 0 \\ 3x + 2a = 0 \rightarrow x = \frac{-2a}{3} \end{cases}$ | نسبی min

طبق فرض $\rightarrow f\left(\frac{-2a}{3}\right) = 0 \rightarrow \left(\frac{-2a}{3}\right)^3 + a\left(\frac{-2a}{3}\right)^2 + 4 = 0$

$\rightarrow -\frac{8a^3}{27} + \frac{4a^3}{9} + 4 = 0 \rightarrow a^3 = -27 \rightarrow a = -3$ |

x نسبی min $= \frac{-2a}{3} = \frac{-2(-3)}{3} = 2$

$f(x) = x^4 - 4x^3 + 5 \Rightarrow f'(x) = 4x^3 - 12x^2 = 0 \rightarrow 4x^2(x - 3) = 0 \rightarrow \begin{cases} x = 0 \text{ نسبی max} \\ x = 3 \checkmark \text{ (A)} \\ x = -3 \checkmark \text{ (B)} \end{cases}$

$f''(x) = 12x^2 - 12 = 0 \rightarrow x = \begin{cases} 1 & \text{(C)} \\ -1 & \text{(D)} \end{cases}$

x	-3	0	3
$f'(x)$	$-$	$+$	$-$
$f(x)$	↓	↑	↓
	min نسبی	max نسبی	min نسبی

$f(-3) = -4 \rightarrow A$ | -3 / -4

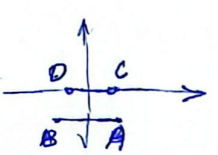
$f(3) = -4 \rightarrow B$ | 3 / -4

$f(1) = 0 \rightarrow C$ | 1 / 0

$f(-1) = 0 \rightarrow D$ | -1 / 0

$\rightarrow m_{AB} = \frac{-4-(-4)}{3+3} = 0$

$\rightarrow m_{CD} = 0$



$\Rightarrow$ زاویه بین دو خط موازی است با صفر