$f(1) = 1 - a, f(e) = 1 - \frac{a}{4} \Rightarrow m = \frac{1 - \frac{a}{4} - (1-a)}{e-1} = \frac{\frac{3a}{4}}{e-1} = \frac{a}{4(e-1)}$

$\Rightarrow f'(x) = \frac{a}{x^2} \rightarrow \frac{a}{x^2} = \frac{a}{4} \rightarrow x^2 = 4 \Rightarrow x = \pm 2$ (۲)

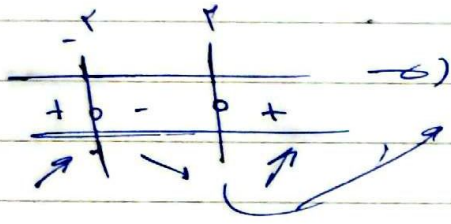
$x = -2$ ✗ ، $x = 2$ ✓

$y = 4ax - 2 = -1 \Rightarrow 4ax = 1 \Rightarrow x = \frac{1}{4a}$ ، $f\left(\frac{1}{4a}\right) = -\frac{1}{a}$

$-\frac{1}{a} = \frac{2}{a} - \frac{2}{a} + \ln a \Rightarrow \ln a^2 = 2 \Rightarrow a^2 = \frac{1}{e^2} \Rightarrow a = \pm\frac{1}{e} \Rightarrow a = \frac{1}{e}$ (۱/۵)

$a = -\frac{1}{e}$ غ ق ق ، $a = \frac{1}{e}$ ← مماس تابع ← $x^2 - 4x + 4 = (x-2)^2 = 0$ ← غ ق ق مثبت

$y = x^3 - 3x + 2 \Rightarrow y' = 3x^2 - 3 \Rightarrow y' = 3(x^2 - 1) = 0 \Rightarrow x = \pm 1$



$x = -1$ ، $y = 4$ (۲)

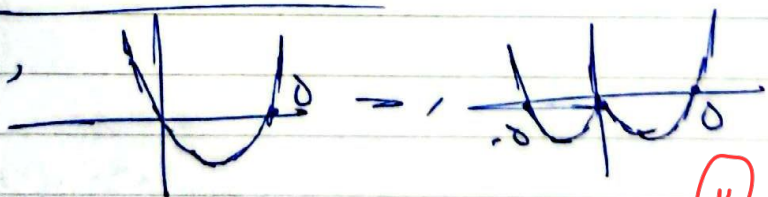
با توجه به نمودار سوال ← $f'(0) = f'(2) = 0$ ✗ (۲)

$y = x^4 + ax^3 - 3bx - 4 \Rightarrow y' = 4x^3 + 3ax^2 - 3b$ ، $x = -1 \Rightarrow -3b = 0 \Rightarrow b = 0$

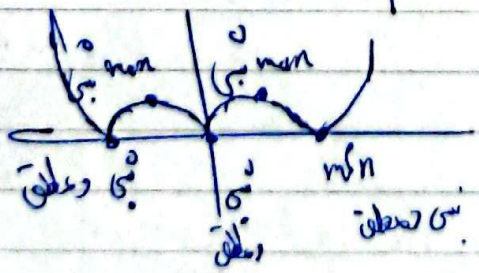
$y = x^4 + x^3 - 4 \Rightarrow y' = 4x^3 + 3x^2 = x^2(4x + 3) = 0 \Rightarrow x = 0, -\frac{3}{4}$

$x = -\frac{3}{4}$ ✓ ، $x = 0$ ✗

$f(x) = x^2 - 2|x| + 1$ ←



←



$\Rightarrow \frac{n}{m} = \frac{2}{1}$ (۲)

D S

$y = |x(|x|+1)| - |x|x|-1|$ ← تمرین

$f'(x) = \begin{cases} 2x+1 & x>0 \\ -2x+1 & x<0 \end{cases}$ → $f'_+(0) = f'_-(0) = 1$

$\begin{cases} x^2+2x & x \geq 0 \\ -x^2+2x & x<0 \end{cases}$ ← تمرین

$|f(x)|$

← تمرین

$f(x) = \sqrt[3]{x^2}|x-a| \rightarrow \sqrt[3]{x^2}(a-x) \rightarrow f'(x) = \frac{2(a-x)}{3\sqrt[3]{x}} - \sqrt[3]{x^2} = 0$

$x \in [0,a] \rightarrow |x-a| = -(x-a) \rightarrow f(x) = -\sqrt[3]{x^2}(x-a) = -x^{\frac{5}{3}} + ax^{\frac{2}{3}}$

$\rightarrow f'(x) = -\frac{5}{3}x^{\frac{2}{3}} + \frac{2}{3}ax^{-\frac{1}{3}} = 0 \rightarrow \frac{1}{3}x^{-\frac{1}{3}}(-5x+2a) = 0 \rightarrow \begin{cases} x=0 \\ x=\frac{2}{5}a \end{cases} \rightarrow max \checkmark$

$f(x_{max}) = 1.2 \rightarrow f(\frac{2}{5}a) = \frac{4}{3} \rightarrow -\sqrt[3]{\frac{4}{25}a^2}(\frac{2}{5}a - a) = \frac{4}{3} \rightarrow ax\sqrt[3]{\frac{4}{25}a^2} = \frac{5}{3}$

$\xrightarrow{\text{به توان } 3} a^3 \times \frac{4}{25}a^2 = \frac{125}{8} \rightarrow a^5 = \frac{125}{8} \times \frac{25}{4} = (\frac{5}{2})^5 \rightarrow a = \frac{5}{2} = 2.5$

$y = \sqrt{x|x| - x} \rightarrow \begin{cases} \sqrt{x^2-x} & x \geq 0 \\ \sqrt{-x^2-x} & x<0 \end{cases}$

$y = \sqrt{x|x| - x}$

$\frac{2x-1}{2\sqrt{x^2-x}}$ ← تمرین

$\frac{-2x-1}{2\sqrt{-x^2-x}}$

$\frac{2x-1}{2\sqrt{x^2-x}} \rightarrow$ 0 | + | 1/2 | − | 1 | +

$\rightarrow x=2, x=1, x=\frac{1}{2}$ و $x=0$

$\frac{2x-1}{2\sqrt{-x^2-x}}$ → −1 | + | −1/2 | − | 0

$x=-1$ و $x=-\frac{1}{2}$ و $x=0$

$\rightarrow$ 1) $\frac{11}{24}$ (تعداد جواب‌ها) $K=4$ ، (max تعداد) $M=1$ ، (min تعداد) $n=0$

$\frac{Km+M}{K-n} = \frac{4\times1-0}{4-0} = \frac{4}{4} = 1$

$(1+\alpha) \in D_f \rightarrow 1-m \neq 1 \rightarrow m \neq 0$ و $y' = \frac{m(m+1)-2}{(x_0 m-1)^2} < 0$

$\rightarrow m^2-m-2 < 0 \rightarrow (m-2)(m+1) < 0 \rightarrow -1 < m < 2$

$\rightarrow 0 < m < 2$ $\xrightarrow{m \in \mathbb{Z}}$ $m=0$ یا $m=1$

تمرین

D

$x \geq 0$ $\quad$ $x < 0$

$D_f = \mathbb{R} - \{1\} \longrightarrow f(x) = \dfrac{x}{1-x^2} \;\; ; \;\; \dfrac{x}{1+x^2}$

$\longrightarrow f'(x) = \begin{cases} \dfrac{(1-x^2)-(-2x)x}{(1-x^2)^2} = \dfrac{x^2+1}{(1-x^2)^2} & x \geq 0 \\ \dfrac{(1+x^2)-(2x)x}{(1+x^2)^2} = \dfrac{1-x^2}{(1+x^2)^2} & x < 0 \end{cases}$

$\longrightarrow f'(x) > 0$ ✓ (2)