14,75

نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس ۱۴ دوازدهم سر A .....

۱

$f'(3) = \frac{4}{3}$

$f(3) = 8$

(۲)

$d : mx + 1$

$(0,1)$

$(3,8)$

$m = \frac{8-1}{3-0} = \frac{4}{3}$ ✓

۲

$A\ (m,n)$

$d : \frac{1}{3}x + \frac{4}{3}$

$y = \frac{x+4}{3}$

$n = \frac{m+4}{3}$

$f'(x) = \frac{a}{2\sqrt{ax-1}} \rightarrow \frac{a}{2\sqrt{am-1}} = \frac{1}{3}$

$f(x) = \sqrt{am-1} = n$

$\frac{m+4}{3} = n$

$\sqrt{am-1} = \frac{m+4}{3}$, $\frac{a}{2\sqrt{am-1}} = \frac{1}{3} \rightarrow \frac{3a}{2} = \sqrt{am-1}$

$\frac{3a}{2} = \frac{m+4}{3} \rightarrow m = \frac{9a-8}{2}$

$\rightarrow \frac{9}{4}a = \sqrt{(\frac{9a-8}{2})a - 1} \rightarrow a = 2$

$f(5) = \sqrt{9} = 3$

(۲)

۳

$y = \frac{2+n}{4}$ $\quad f'(n) = \frac{3}{4}$ $\quad \frac{m+2}{4} = \frac{2+n}{4} \rightarrow n+1 = m$

$f(1) = \frac{m+2}{4}$

$f'(n)_{x=1} = \frac{(2+m)(4) - (1)(2+m)}{16} \rightarrow \frac{3m+6}{16} = \frac{3}{4} \rightarrow m = 2$

$n = 1$ ✓

$m + n = 3$

(۲)

۴

$f(n) = \frac{(3 - \sin n)(9 + \sin n^2 + 3\sin n)}{(3 - \sin n)(3 + \sin n)} = \frac{9 + \sin n^2 + 3\sin n}{3 + \sin n}$

$3g(n) - f(n) = \frac{9}{3+\sin n} - \frac{9 + \sin n^2 + 3\sin n}{3 + \sin n} = \frac{-\sin n^2 - 3\sin n}{3 + \sin n}$

$\frac{-\sin n(\sin n + 3)}{3 + \sin n} = -\sin n \rightarrow (3g(n) - f(n))' = -\cos n$

$-\cos(\frac{2\pi}{3}) = +\frac{1}{2}$

$-\frac{1}{2}$

1,75

۵

$(f \circ g(n))' \Rightarrow \quad f \circ g(n) = -\frac{1}{\sqrt[5]{\frac{1}{x^5 + |x^5|} + \frac{1}{x^5 + |x^5|}}} = -\frac{1}{\sqrt[5]{\frac{1}{2x^5} + \frac{1}{2x^5}}}$

$= -\frac{1}{\sqrt[5]{\frac{1}{x^5}}} = -x \quad (f \circ g(n))' = -1$ ✓

(۲)

۶

$f(x) = x\,g(x) + 1 \longrightarrow \dfrac{f(x)-1}{x} = g(x)$

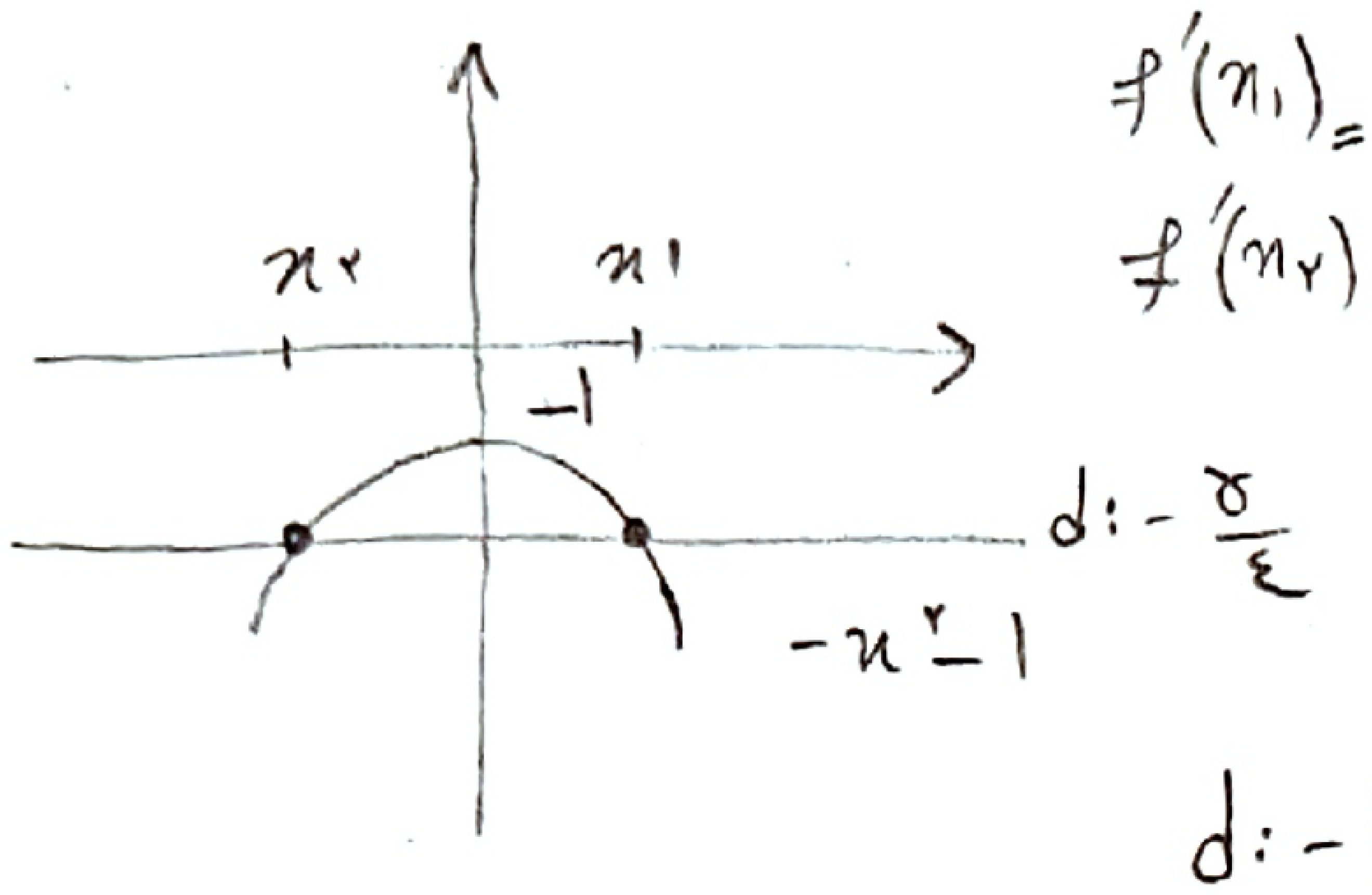
$\lim\limits_{x\to 0} \dfrac{f(x)-1}{x-0} = \lim g(x)$

$f'(x) = 2\left(\dfrac{\sin x - 1}{\sin x + 1}\right)\left(\dfrac{\cos x(1) + \cos x(1)}{(\sin x + 1)^2}\right) = 2 \times -1 \times 2 = -4$

$\lim\limits_{x\to 0} g(x) = -4$

(۲)

۷



$f'(x_1) = -2x_1$

$f'(x_2) = -2x_2 \longrightarrow \{4x_1x_2 = -1$

$x_1x_2 = -\frac{1}{4}$

$x_1 = -x_2$

$x_1 = \frac{1}{2}\ ,\ x_2 = -\frac{1}{2}$

$d: -\frac{5}{4}$

پس خط d برابر $\frac{5}{4}$ است.

(۲)

۸

$d: mx \longrightarrow (0,0),\ (a,\ 2\sqrt{a}\,(4a^2+3)) \qquad m = 8\sqrt{2}$

$f'(a) = m$

$\dfrac{f(a)-0}{a-0} = \dfrac{f(a)}{a} = f'(a) \longrightarrow f'(x) = 2\dfrac{1}{2\sqrt{x}}(4x^2+3) + 8x \times 2\sqrt{x}$

$= \dfrac{20x^2+3}{\sqrt{x}} \xrightarrow{a=\sqrt{\frac{1}{2}}} \dfrac{20a^2+3}{\sqrt{a}} = \dfrac{2(4a^2+3)}{\sqrt{a}}$

$20a^2 + 3 = 8a^2 + 6 \longrightarrow a^2 = \frac{1}{4}\ ,\ a \geq 0$

(۲)

۹

$d: mx \longrightarrow (0,0),\ \left(a,\ \dfrac{\sqrt{a}}{-2a^2+a+1}\right) \qquad f\left(\frac{1}{2}\right) = \dfrac{\sqrt{\frac{1}{2}}}{-\frac{2}{4}+\frac{2}{4}+1} = \sqrt{\frac{1}{2}}$

$f'(a) = m$

$\dfrac{f(a)-0}{a-0} = \dfrac{f(a)}{a} = f'(a) \longrightarrow f'(x) = \dfrac{6x^2 - x + 1}{2(-2x^2+x+1)^2\sqrt{x}}$

$\dfrac{\sqrt{a}}{a(-2a^2+a+1)} = \dfrac{6a^2 - x + 1}{(-2a^2+a+1)^2\sqrt{a}} \longrightarrow 6a^2 - a + 1 = -4a^2 + 2a + 2$

$10a^2 - 3a - 1 = 0 \longrightarrow a = \frac{1}{2}, -\frac{1}{5}$

(۲)

۱۰

$(f \circ g(x))' \qquad f(g(x)) = \left(2 \times \dfrac{1}{\sqrt{x^2-1}}\right)^3$

$g(x) = \dfrac{1}{\sqrt{\frac{5}{4}-1}} = 2 \qquad 3\left(2 \times \dfrac{1}{\sqrt{x^2-1}}\right)^2 \times 2\ \dfrac{-\frac{2x}{2\sqrt{x^2-1}}}{x^2-1}$

$-3(2\times 2)^2 \times 2 \times \dfrac{\frac{\sqrt{5}/2}{1/2}}{\frac{1}{4}} = -3 \times 16 \times 2 \times 4\sqrt{5} = -48\sqrt{5} \times 8$

۸ برابر است.

(۲)