

$$f(x) = \frac{r}{p}x + 1$$

$$f'(x) = \frac{r}{p}$$

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$$y = ax + b \begin{cases} 1 = -a + b \\ r = 2a + b \end{cases} \rightarrow -1 = -2a \rightarrow a = \frac{1}{2}, b = \frac{r}{2}$$

$$y = \frac{1}{2}x + \frac{r}{2}$$

$$f(x) = y \rightarrow \sqrt{ax-1} = \frac{x+r}{r}$$

$$f(x) = \sqrt{2x-1} \xrightarrow{x=0} \sqrt{1} = 1$$

$$x^2 + (1-2a)x + 2b = 0$$

$$\Delta = 0 \rightarrow a = \frac{1}{2}$$

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$\frac{r}{9}x$

$$y = \frac{r}{f}x - \frac{n}{f}$$

$$\frac{x^2 + mx + 1}{x+r}$$

$$x=1 \rightarrow \frac{r+n}{f} = \frac{r+m}{f} \rightarrow m = n+1$$

$$\frac{r}{f} = \frac{(r+n)(r) - (1)(x^2 + mx + 1)}{(x+r)^2}$$

$$\rightarrow \frac{r}{f} = \frac{(r+n)(r) - (1)(r^2)}{1} \rightarrow \frac{r(r+n)}{1} = \frac{r^2}{f} \rightarrow r(r+n) = \frac{r^2}{f} \rightarrow m = 2$$

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$$f(x) = \frac{(r - \sin x)(9 + \sin^2 x + \sin x)}{(r - \sin x)(r + \sin x)} = \frac{\sin^2 x + r \sin x + 9}{r + \sin x}$$

$$g(x) = \frac{9}{r + \sin x}$$

$$(rg - f)'(x) = \frac{-\sin^2 x - r \sin x}{r + \sin x} \xrightarrow{\text{مستقیماً بگیریم}} \frac{-(\cos x)(r - \sin x)}{(r - \cos x)} = \cos x$$

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$$f(x) = \frac{1}{\sqrt{2x}}$$

$$g(x) = \frac{1}{2x}$$

$$(f \circ g)(x) = \frac{-1}{\sqrt{\frac{1}{2x}}} = \frac{-1}{\frac{1}{\sqrt{2x}}} = -\sqrt{2x}$$

$$(f \circ g)'(\sqrt{r}) = -1$$

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$$g(x) = \frac{f(x)-1}{x} = \frac{\left(\frac{-1+\sin x}{1+\sin x}\right)^2 - 1}{x}$$

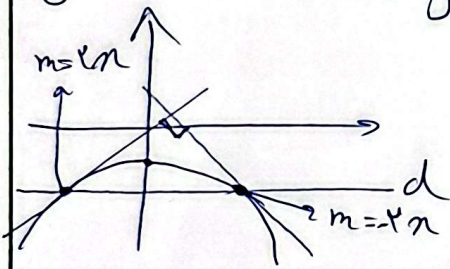
$$g'(x) = \frac{2 \times \left(\cos x (1+\sin x) - (-1+\sin x) \cos x \right) \times \left(\frac{-1+\sin x}{1+\sin x} \right)}{x^2}$$

$$g'(x) = \frac{2 \times 2 \times x - 1}{x^2} = \frac{4x-1}{x^2}$$

منهج كارتان
 $(\sin x + 1)^2$

$$y = -x^2 - 1 \rightarrow y' = -2x$$

$\frac{dy}{dx} = -2x$



$$2x \times (-2x) = -1 \rightarrow -4x^2 = -1$$

$$-x^2 = -\frac{1}{4} \rightarrow x = \pm \frac{1}{2}$$

$$f(x) = \lambda x^2 \sqrt{x} + 4\sqrt{x} = mx \rightarrow \lambda x^2 + 4 = m\sqrt{x}$$

$$f'(x) = 2\lambda x = \frac{m}{2\sqrt{x}} \rightarrow m = 4\lambda x \sqrt{x}$$

$$\lambda x^2 + 4 = 4\lambda x \sqrt{x} \rightarrow \lambda x^2 = 4\lambda x \sqrt{x} - 4 \rightarrow x^2 = 4x\sqrt{x} - \frac{4}{\lambda} \rightarrow x = \pm \frac{1}{2}$$

$$\lambda = \frac{m\sqrt{x}}{x} \rightarrow m = \lambda \sqrt{x}$$

$$\frac{1}{-2x^2 + x + 1} = m\sqrt{x} \rightarrow \frac{-2x^2 + x + 1}{2x^2 - x - 1} = \frac{m\sqrt{x}}{x} = 2x$$

$$\frac{-(-2x+1)}{(2x^2+x+1)^2} = \frac{m}{2\sqrt{x}}$$

$$A = \frac{1}{\sqrt{x}}$$

$$\lambda x^2 - 2x = -2x^2 + x + 1 \rightarrow \lambda x^2 - 2x - 1 = 0$$

$$x \geq 0 \rightarrow \lambda x^2 + 2x - 1 = 0 \rightarrow x = \frac{1}{2}$$

$$y = \frac{\sqrt{x}}{x}$$

$$f(x) = (2x)^2$$

$$(f \circ g)'(x) = (2\sqrt{2}) \times 2x = \frac{-2\sqrt{2}}{-2\sqrt{2}} = 1$$