تکلیف ۲۴ — دوازدهم ۲ A — برنیا براتی

۱-

$$y - 5 = \frac{5-1}{3-0}(x-0) \Rightarrow y = \frac{4}{3}x + 5$$

$$\Rightarrow f'(3) = \frac{4}{3}$$

۲-

$$\frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{2-1}{2+1} = \frac{1}{3} \Rightarrow f'(x) = \frac{a}{2\sqrt{ax-1}} = \frac{1}{3} \Rightarrow 3a = 2\sqrt{ax-1}$$

$$9a^2 = 4ax - 4 \Rightarrow 9a^2 - 4ax + 4 = 0 \Rightarrow ax_0 = \frac{9a^2+4}{4}$$

$$y - y_0 = \frac{1}{3}(x - x_0), \quad y_0 = \sqrt{ax_0 - 1} = \sqrt{\frac{9a^2}{4}}$$

$$\Rightarrow y_0 = \frac{3a}{2}, \quad x_0 = \frac{9a^2+4}{4a}$$

$$2 - \frac{3a}{2} = \frac{1}{3}(2 - x_0) \Rightarrow x_0 = \frac{9a-8}{2} \Rightarrow \frac{9a-8}{2} = \frac{9a^2+4}{4a}$$

$$\Rightarrow 9a^2 - 16a - 4 = 0 \Rightarrow a = 2 \checkmark, \ -\frac{2}{9} \times \Rightarrow a = 2 \Rightarrow f(5) = \sqrt{2 \times 5 - 1}$$

$$= 3$$

۳-

$$\frac{3+n}{4} = \frac{2+m}{4} \Rightarrow 3 + n = 2 + m$$

$$y' = \frac{x^2 + 6x + 3m - 1}{(x+3)^2} \Rightarrow f'(1) = \frac{6+3m}{16} = \frac{3}{4} \Rightarrow m = 2$$

$$\Rightarrow n = 1 \Rightarrow m + n = 2 + 1 = 3$$

۴-

$$f(x) = \frac{(3 - \sin x)(9 + \sin^2 x + 3\sin x)}{(3 - \sin x)(3 + \sin x)} = \frac{9 + \sin^2 x + 3\sin x}{3 + \sin x}$$

$$3g'(x) - f'(x) = (3g(x) - f(x))' = \left(\frac{9}{3+\sin x} - \frac{9 + \sin^2 x + 3\sin x}{3 + \sin x}\right)'$$

$$= (-\sin x)' = -\cos x \Rightarrow 3g'\left(\frac{5\pi}{3}\right) - f'\left(\frac{5\pi}{3}\right) = -\frac{1}{2}$$

۵-

$$-\frac{1}{\sqrt[5]{\frac{1}{x^5+|x^5|}+\left|\frac{1}{x^5+|x^5|}\right|}} = f\circ g(x) \Rightarrow f\circ g(\sqrt[5]{2}) = \frac{-1}{\sqrt[5]{\frac{1}{x^5}}}$$

$$\Rightarrow f\circ g(\sqrt[5]{2}) = -x \Rightarrow f\circ g'(\sqrt[5]{2}) = -1$$

۶-

$$x g(x)+1 = \frac{\sin^2 x+1-2\sin x}{\sin^2 x+1+2\sin x} \Rightarrow g(x) = \frac{\frac{-4\sin x}{\sin^2 x+1+2\sin x}}{x}$$

$\sin x \sim x$

$$\Rightarrow g(x) = \frac{\frac{-4x}{x^2+1+2x}}{x} = \frac{-4}{x^2+1+2x} \Rightarrow \lim_{x\to 0} g(x) = -4$$

۷-

$$f(x) = -x^2-1 \Rightarrow f'(x) = -2x \Rightarrow -2x_1 = \frac{1}{2x_2}$$

$$, x_1 = -x_2 \Rightarrow 4x_1^2 = 1 \Rightarrow x_1 = +\frac{1}{2} , x_2 = -\frac{1}{2}$$

$$y = -\frac{1}{4}-1 = -\frac{5}{4} \Rightarrow$$ طول d از محور $x = \frac{5}{4}$

۸-

$$m_d = f'(x) = \frac{4x^2+3}{\sqrt{x}} + 16x\sqrt{x} \Rightarrow f'(x) = \frac{20x^2+3}{\sqrt{x}}$$

$$f'(x) = \frac{f(x)-0}{x} \Rightarrow \frac{2\sqrt{x}(4x^2+3)}{x} = \frac{20x^2+3}{\sqrt{x}} \Rightarrow x^2 = \frac{1}{4}$$

$x>0$

$$\Rightarrow x = +\frac{1}{2} \checkmark , -\frac{1}{2} \times \Rightarrow x = \frac{1}{2}$$

۹-

$$f'(x) = \frac{\frac{6x^2-x+1}{2\sqrt{x}}}{(-2x^2+x+1)^2} = \frac{\frac{\sqrt{x}}{-2x^2+x+1}}{x} = \frac{6x^2-x+1}{2\sqrt{x}(-2x^2+x+1)} = \frac{\sqrt{x}}{x}$$

$$\Rightarrow 1, x^2-4x-1=0 \Rightarrow x=5 \Rightarrow y = \frac{\sqrt{5}}{-46}$$

۱۰-

$$(f\circ g)'(x) = \frac{x}{(x^2-1)\sqrt{x^2-1}} \times f'(g(x)) , f'(x) = 3(x[x])^2 \times [x]$$

$$g\left(\frac{\sqrt{5}}{2}\right) = 2^+ \Rightarrow f'(2^+) = 96 \Rightarrow f\circ g'\left(\frac{\sqrt{5}}{2}\right) = 4\sqrt{5}\times 96$$

$$\Rightarrow \frac{4\sqrt{5}\times 96}{-48\sqrt{5}} = -8$$