۲۴ تکلیف — دوازدهم A

$y - 5 = \frac{5-1}{3-0}(x-0) \Rightarrow y = \frac{4}{3}x + 5$

$\Rightarrow f'(3) = \frac{4}{3}$ ✓

(۲)

۲-

$\frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{2-1}{2+1} = \frac{1}{3} \Rightarrow f'(x) = \frac{a}{2\sqrt{ax-1}} = \frac{1}{3} \Rightarrow 3a = 2\sqrt{ax-1}$

$9a^2 = 4ax - 4 \Rightarrow 9a^2 - 4ax + 4 = 0 \Rightarrow ax_0 = \frac{9a^2+4}{4}$

$y - y_0 = \frac{1}{3}(x - x_0), \quad y_0 = \sqrt{a x_0 - 1} = \sqrt{\frac{9a^2}{4}}$

$\Rightarrow y_0 = \frac{3a}{2}, \quad x_0 = \frac{9a^2+4}{4a}$

(۲)

$2 - \frac{3a}{2} = \frac{1}{3}(2 - x_0) \Rightarrow x_0 = \frac{9a-8}{2} \Rightarrow \frac{9a-8}{2} = \frac{9a^2+4}{4a}$

$\Rightarrow 9a^2 - 16a - 4 = 0 \Rightarrow a = 2, -\frac{2}{9} \Rightarrow a = 2 \Rightarrow f(5) = \sqrt{2 \times 5 - 1}$

$= 3$ ✓

۳-

$\frac{3+n}{4} = \frac{2+m}{4} \Rightarrow 3+n = 2+m$

$y' = \frac{x^2+6x+3m-1}{(x+3)^2} \Rightarrow f'(1) = \frac{6+3m}{16} = \frac{3}{4} \Rightarrow m = 2$

$\Rightarrow n = 1 \Rightarrow m+n = 2+1 = 3$ ✓

(۲)

۴-

$f(x) = \frac{(3-\sin x)(9+\sin^2 x+3\sin x)}{(3-\sin x)(3+\sin x)} = \frac{9+\sin^2 x+3\sin x}{3+\sin x}$

$3g'(x) - f'(x) = (3g(x) - f(x))' = \left(\frac{9}{3+\sin x} - \frac{9+\sin^2 x+3\sin x}{3+\sin x}\right)'$

$= (-\sin x)' = -\cos x \Rightarrow 3g'\left(\frac{5\pi}{3}\right) - f'\left(\frac{5\pi}{3}\right) = -\frac{1}{2}$ ✓

(۲)

۵-

$$-\frac{1}{\sqrt[5]{\frac{1}{x^5+|x^5|}+\left|\frac{1}{x^5+|x^5|}\right|}} = f\circ g(x) \Rightarrow f\circ g(\sqrt[5]{2}) = \frac{-1}{\sqrt[5]{\frac{1}{x^5}}}$$

$$\Rightarrow f\circ g(\sqrt[5]{2}) = -x \Rightarrow f\circ g'(\sqrt[5]{2}) = -1$$ ✓ (۲)

۶-

$$x g(x)+1 = \frac{\sin^2 x+1-2\sin x}{\sin^2 x+1+2\sin x} \Rightarrow g(x) = \frac{\frac{-4\sin x}{\sin^2 x+1+2\sin x}}{x}$$ (۲)

$\sin x \sim x$

$$\Rightarrow g(x) = \frac{\frac{-4x}{x^2+1+2x}}{x} = \frac{-4}{x^2+1+2x} \Rightarrow \lim_{x\to 0} g(x) = -4$$ ✓

۷-

$$f(x) = -x^2-1 \Rightarrow f'(x) = -2x \Rightarrow -2x_1 = \frac{1}{2x_2}$$

$$, x_1 = -x_2 \Rightarrow 4x_1^2 = 1 \Rightarrow x_1 = +\frac{1}{2}, x_2 = -\frac{1}{2}$$

$$y = -\frac{1}{4}-1 = -\frac{5}{4} \Rightarrow$$ طول d از محور x ها $= \frac{5}{4}$ (۲)

۸-

$$m_d = f'(x) = \frac{4x^2+3}{\sqrt{x}} + (6x\sqrt{x}) \Rightarrow f'(x) = \frac{6x^2+3}{\sqrt{x}}$$ (۱٫۵)

$$f'(x) = \frac{f(x)-0}{x} \Rightarrow \frac{2\sqrt{x}(4x^2+3)}{x} = \frac{20x^2+3}{\sqrt{x}} \Rightarrow x^2 = \frac{1}{4}$$

$x>0$

$$\Rightarrow x = +\frac{1}{2}, -\frac{1}{2} \Rightarrow x = \frac{1}{2}$$

$$m = \frac{20(\frac{1}{4})+3}{\sqrt{\frac{1}{2}}} = 8\sqrt{2}$$

۹-

$$f'(x) = \frac{\frac{6x^2-x+1}{2\sqrt{x}}}{(-2x^2+x+1)^2} = \frac{\sqrt{x}}{\frac{-2x^2+x+1}{x}} = \frac{6x^2-x+1}{2\sqrt{x}(-2x^2+x+1)} = \frac{\sqrt{x}}{x}$$ (۱٫۵)

$$f(\alpha) = \frac{\sqrt{\frac{1}{2}}}{-2(\frac{1}{2})^2+\frac{1}{2}+1} = \frac{\sqrt{2}}{2}$$

$$\Rightarrow 10x^2-3x-1 = 0 \Rightarrow x = 5 \Rightarrow y = \frac{\sqrt{5}}{-46}$$

$\alpha = \frac{-1}{5}$, $\alpha = \frac{1}{2}$

۱۰-

$$(f\circ g)'(x) = \frac{x}{(x^2-1)\sqrt{x^2-1}} \times f'(g(x)), \quad f'(x) = 3(x[x])^2 \times [x]$$

$$g\left(\frac{\sqrt{5}}{2}\right) = 2^+ \Rightarrow f'(2^+) = 96 \Rightarrow f\circ g'\left(\frac{\sqrt{5}}{2}\right) = -4\sqrt{5}\times 96$$ (۱٫۷۵)

$$\Rightarrow \frac{-4\sqrt{5}\times 96}{-48\sqrt{5}} = +8$$