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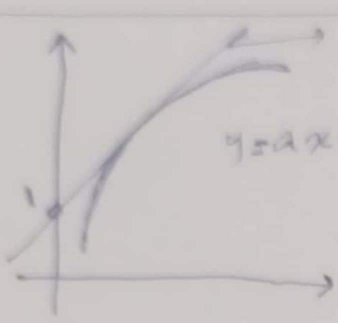
نام و نام خانوادگی ......فریبا غفاری...... پاسخنامه تشریحی تکلیف شماره ...... کلاس ....



$\frac{4}{3}x+1$

$y=ax+1 \Rightarrow 3a+1=5 \Rightarrow 3a=4 \Rightarrow a=\frac{4}{3}$

$f'(3)=\frac{4}{3}$ مشتق تابع برابر است با شیب خط مماس

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$m=\frac{2-(+1)}{2-(-1)}=\frac{1}{3}$

$f(x)=\sqrt{ax-1} \xrightarrow[\text{خط مماس}]{\text{شیب}} f'(x)=\frac{a}{2\sqrt{ax-1}} \rightarrow \frac{a}{2\sqrt{ax-1}}=\frac{1}{3} \rightarrow 3a=2\sqrt{ax-1}$ (I)

معادله خط گذرنده از ۲ نقطه $= y=\frac{1}{3}x+\frac{4}{3} \rightarrow 3y=x+4 \rightarrow x+4=3\sqrt{ax-1}$ (II)

$\frac{1}{3}x+b \Rightarrow \frac{1}{3}(2)+b=2 \Rightarrow \frac{2}{3}+b=2 \Rightarrow b=\frac{4}{3}$ — $f(5)=\sqrt{2(5)-1}=\sqrt{4}=3$ — ۲

$y=\frac{1}{3}x+\frac{4}{3}$

$\xrightarrow{I,\ II} x+4=(\frac{3a}{2})^2=\frac{9a}{2} \rightarrow x=4.5a-4$

$\xrightarrow{II} 4.5a-4+4=3\sqrt{a(4.5a-4)-1} \rightarrow 9a^2-16a-4=0 \rightarrow \begin{cases} a=2 \\ a=-\frac{2}{9} \end{cases}$

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$4y-3x=n \Rightarrow 4y=3x+n \Rightarrow y=\frac{3}{4}x+\frac{n}{4}$

$\frac{(2x+m)(x+3)-((x^2+mx+1)(1))}{(x+3)^2}=\frac{((2+m)\times4)-(2+m)}{16}=\frac{3}{4}$ — ۳

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$\frac{6+3m}{16}=\frac{3}{4} \Rightarrow 24+12m=48$

$12m=24 \Rightarrow m=2$

$f(x)=\frac{x^2+2x+1}{x+3} \xrightarrow{x=1} \frac{4}{4}=1$

$m+n=1+2=3$ ✓

$\frac{3}{4}(1)+\frac{n}{4}=1 \Rightarrow n=1$

$3g'(\frac{5\pi}{3})-f'(\frac{5\pi}{3})$

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$f(x)=\frac{27-\sin^3x}{9-\sin^2x} \Rightarrow f'(x)=\frac{(-3\cos^2x\times(9-\sin^2x))-(-2\cos x\times(27-\sin^3x))}{(9-\sin^2x)^2}$ — ۴

(جواب پایین صفحه)

$g(x)=\frac{3}{3+\sin x} \Rightarrow g'(x)=\frac{0-(3\times\cos x)}{(3+\sin x)^2} \stackrel{x=\frac{5\pi}{3}}{=} \frac{-3\times\frac{1}{2}}{(3-\frac{\sqrt{3}}{2})^2}=\frac{-\frac{3}{2}}{9-3\sqrt{3}+\frac{3}{4}}$

$g(x)=\frac{1}{2x^5} \Rightarrow g'(x)=\frac{-10x^4}{4x^{10}}=\frac{-10}{4x^6}=\frac{-10}{4\times3\sqrt[5]{3}}=\frac{-10}{12\sqrt[5]{3}}=\frac{-5}{6\sqrt[5]{3}}$

(جواب پایین صفحه)

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$f(x)=\frac{1}{\sqrt[5]{2x}} \Rightarrow f'(x)=\frac{-\frac{2}{5\sqrt[5]{16x^4}}}{\sqrt[5]{4x^2}}=\frac{\frac{-2}{5\sqrt[5]{16\times\sqrt[5]{81}}}}{\sqrt[5]{4\sqrt[5]{9}}}=\frac{-2}{5\sqrt[5]{16\times\sqrt[5]{81}}\times\sqrt[5]{4\sqrt[5]{9}}}=\frac{-2}{5\sqrt[5]{64\times\sqrt[5]{3}}}$

$g'(x)f'(x)=\frac{-5}{6\sqrt[5]{3}}\times\frac{-2}{5\sqrt[5]{64\times3\sqrt[5]{3}}}=\frac{10}{6\sqrt[5]{3}\times10\sqrt[5]{2\times3\sqrt[5]{3}}}=\boxed{\frac{1}{6\sqrt{18\sqrt[5]{3}}}}$

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$f(x) = x\,g(x)+1 \rightarrow g(x) = \frac{f(x)-1}{x} \rightarrow \lim_{x\to 0} g(x) = \frac{0}{0} \xrightarrow{hop} \frac{f'(0)}{1} = f'(0)$

$f(x) = \left(\frac{-1+\sin x}{1+\sin x}\right)^2 \rightarrow f'(x) = 2\left(\frac{\cos x(1+\sin x) - \cos x(-1+\sin x)}{(1+\sin x)^2}\right) \times \left(\frac{-1+\sin x}{1+\sin x}\right)$ ۶

$\rightarrow f'(0) = 2\times\left(\frac{2}{1}\right)\times(-1) = -4$

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$y = -x^2 - 1$

$y = x^2+1 \xrightarrow[\text{محور } x]{\text{قرینه نسبت به}} y_1 = -(x^2+1) = -x^2-1 \xrightarrow{\text{مشتق}} y' = -2x$

خط d، منحنی y_1 را در نقاط A و B قطع کند چون خط d موازی محور xها است ← نقاط A و B ← عرض یکسان و طول قرینه دارند.

$A(\alpha, \beta)\ ,\ B(-\alpha, \beta) \longrightarrow A\left(\frac{1}{2}, \beta\right)\ ,\ B\left(-\frac{1}{2}, \beta\right)$ ۷

$mL_1 = y_1'(-\alpha) = -2(-\alpha) = 2\alpha$

$mL_2 = y_1'(\alpha) = -2\alpha$

L_1, L_2 عمود اند $\longrightarrow mL_1 \times mL_2 = -1 \longrightarrow -2\alpha\cdot 2\alpha = -1 \rightarrow 4\alpha^2 = 1 \rightarrow \alpha = \pm\frac{1}{2}$

فاصله خط از مبدا مختصات $\rightarrow |\beta| \longrightarrow \beta = y_1\left(\frac{-1}{2}\right) = -\left(-\frac{1}{2}\right)^2 - 1 = -\frac{1}{4} - 1 = \frac{-5}{4} = -1{,}25 \longrightarrow |\beta| = 1{,}25$ ۰

$f(x) = 2\sqrt{x}(4x^2+3) \Rightarrow f'(x) = 2\times\frac{1}{2\sqrt{x}}\times(4x^2+3) + 8x\times 2\sqrt{x}$

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$f(x) = 2\sqrt{x}(4x^2+3) = 8x^2\sqrt{x} + 6\sqrt{x} \longrightarrow f'(x) = 20x\sqrt{x} + \frac{3}{\sqrt{x}} = \frac{20x^2+3}{\sqrt{x}}$

$y - 2\sqrt{\alpha}(4\alpha^2+3) = \frac{20\alpha^2+3}{\sqrt{\alpha}}(x-\alpha) \xrightarrow{(0,0)} -2\sqrt{\alpha}(4\alpha^2+3) = \frac{20\alpha^2+3}{\sqrt{\alpha}}(-\alpha)$ ۸

$\rightarrow 2(4\alpha^2+3) = 20\alpha^2+3 \rightarrow 12\alpha^2 = 3 \rightarrow \alpha^2 = \frac{1}{4}$

$m = \frac{20\left(\frac{1}{4}\right)+3}{\sqrt{\frac{1}{2}}} = 8\sqrt{2}$

$f(x) = \frac{\sqrt{x}}{-2x^2+x+1} \Rightarrow f'(x) = \frac{\frac{1}{2\sqrt{x}}\times(-2x^2+x+1) - (-4x+1)\times(\sqrt{x})}{(-2x^2+x+1)}$ ۰/۸

خط d $\rightarrow y = ax$ $\qquad A(\alpha, a\alpha)$

$f(\alpha) = \frac{\sqrt{\alpha}}{-2\alpha^2+\alpha+1} = a\alpha \rightarrow a\sqrt{\alpha}(-2\alpha^2+\alpha+1) = 1 \rightarrow -2a\alpha^{\frac{5}{2}} + a\alpha^{\frac{3}{2}} + a\alpha^{\frac{1}{2}} = 1$ ۹

$\xrightarrow{\text{مشتق}} -5a\alpha^{\frac{3}{2}} + \frac{3}{2}a\alpha^{\frac{1}{2}} + \frac{1}{2}a\alpha^{-\frac{1}{2}} = 0 \xrightarrow[\times 2\sqrt{\alpha}]{\div a} -10\alpha^2 + 3\alpha + 1 = 0 \longrightarrow \begin{cases} \alpha = \frac{-1}{5} \\ \alpha = \frac{1}{2} \end{cases}$

$f(\alpha) = \frac{\sqrt{\frac{1}{2}}}{-2\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 1} = \frac{\sqrt{2}}{2}$

$g(x) = \frac{1}{\sqrt{x^2-1}} = \frac{1}{\sqrt{\frac{5}{4} - \frac{1}{4}}} = \frac{1}{\sqrt{\frac{1}{4}}} = \frac{1}{\frac{1}{2}} = 2 \qquad g'(x) = \frac{2x}{2\sqrt{x^2-1}} = \frac{2\times\frac{\sqrt{5}}{2}}{2\times\frac{1}{2}} = \sqrt{5}$

$(f\circ g)'\left(\frac{\sqrt{5}}{2}\right) = g'\left(\frac{\sqrt{5}}{2}\right)\times f'\Big(\underbrace{g\left(\frac{\sqrt{5}}{2}\right)}_{2}\Big) = \boxed{96\sqrt{5}}$ ۱۰

$24\times 4 = 96$

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$f'(2^+) = (2x)^3 = 8x^3 \Rightarrow 24x^2$ $\qquad$ (جواب پایین صفحه)

سوال ۱۴

$$3g'\left(\frac{5\pi}{3}\right) - f'\left(\frac{5\pi}{3}\right) = \left(3g(x) - f(x)\right)'\left(\frac{5\pi}{3}\right)$$

$$\to (3g-f)(x) = \left(\frac{9}{3+\sin x} - \frac{27-\sin^3 x}{9-\sin^2 x}\right) = \frac{9}{3+\sin x} - \frac{(3-\sin x)(9+\sin^2 x+3\sin x)}{(3-\sin x)(3+\sin x)} = -\sin x$$

$$\to (3g-f)'(x) = -\cos x \to (3g-f)'\left(\frac{5\pi}{3}\right) = -\cos\left(\frac{5\pi}{3}\right) = \frac{-1}{2}$$

سوال ۱۵

$$g'(x) \times f'(g(x)) = (f\circ g)'(x)$$

$$x>0 \to g(x) = \frac{1}{2x^5} \;,\; x>0 \to f(x) = \frac{-1}{\sqrt[5]{2x}} \xrightarrow{x>0} f\circ g(x) = \frac{-1}{\sqrt[5]{2\left(\frac{1}{2x^5}\right)}}$$

$$\to f\circ g(x) = -x \to (f\circ g)'(x) = -1 \to (f\circ g)'(\sqrt[5]{3}) = -1$$

سوال ۱۰

$$\left(f\circ g\left(\frac{\sqrt5}{2}\right)\right)' = g'\left(\frac{\sqrt5}{2}\right)^- \times f'\left(g\left(\frac{\sqrt5}{2}\right)^-\right)$$

$$g(x) = (x^2-1)^{-\frac12} \to g'(x) = \frac{-1}{2}(x^2-1)^{-\frac32}\times 2x \to g'\left(\frac{\sqrt5}{2}^-\right) = \frac{1}{\sqrt{\left(\frac54\right)^- - 1}} = \frac{1}{\sqrt{\left(\frac14\right)^-}} = \frac{1}{\left(\frac12\right)^-} = 2^+$$

$$f'(2^+) = \left((2x)^3\right)' = (8x^3)' = 24x^2 = 24\times 4$$

$$\to g'\left(\frac{\sqrt5}{2}^-\right)\times f'\left(g\left(\frac{\sqrt5}{2}\right)^-\right) = -4\sqrt5 \times 4\times 24 \to \frac{24\times 4\times(-4\sqrt5)}{-48\sqrt5} = 8$$