SUBJECT

Year: Month: Day:

A گروه دوازدهم سپیده — تکلیف شماره ۲۴ — ۱۸٫۵ — آرا آزادینا

(۱) $A(0,1)\,,\, B(3,5) \Rightarrow f'(3) = \frac{\Delta y}{\Delta x} = \frac{5-1}{3-0} = \frac{4}{3}$ ✓ (۲)

(۲) $f(x)=\sqrt{ax-1}\,,\, A(-1,1)\,,\, B(2,2) \xrightarrow{\text{جایگذاری}} ax+b=y \Rightarrow y=\frac{1}{3}x+\frac{4}{3}$

$\Rightarrow \sqrt{ax-1}=\frac{x+4}{3} \Rightarrow x^2+8x+16=9ax-9 \Rightarrow x^2+(8-9a)x+25=0$

$\xrightarrow{\Delta=0 \text{ باید}} (8-9a)^2-100=0 \Rightarrow (8-9a)^2=100 \rightarrow 8-9a=10 \Rightarrow a=-\frac{2}{9}$ (غ ق)

$\hookrightarrow 8-9a=-10 \Rightarrow a=2$ (ق ق)

$f(x)=\sqrt{2x-1} \Rightarrow f(5)=3$ ✓ (۲)

(۳) $f(x)=\frac{x^2+mx+1}{x+3} \Rightarrow f'(x)=\frac{(2x+m)(x+3)-(x^2+mx+1)}{(x+3)^2}\,,\, y=\frac{3}{4}x+\frac{n}{4}$ (۲)

$\Rightarrow f'(1)=\frac{3}{4}=\frac{(2+m)(4)-(2+m)}{16} \Rightarrow 3(2+m)=12 \Rightarrow m+2=4 \Rightarrow m=2$

$\{m+n=3$

$f(x)=\frac{x^2+2x+1}{x+3} \Rightarrow f(1)=\frac{4}{4}=1 \Rightarrow 1=\frac{3}{4}+\frac{n}{4} \Rightarrow \frac{3+n}{4}=1 \Rightarrow n=1$ ✓

(۴) $f(x)=\frac{27-\sin^3 x}{9-\sin^2 x}=\frac{(3-\sin x)(\sin^2 x+3\sin x+9)}{(\sin x+3)(3-\sin x)}=\frac{\sin^2 x+3\sin x+9}{3+\sin x}$ (۲)

$g(x)=\frac{3}{3+\sin x} \Rightarrow 3g(x)=\frac{9}{3+\sin x} \Rightarrow 3g(x)-f(x)=\frac{9-\sin^2 x-3\sin x-9}{3+\sin x}$

$=\frac{-(\sin x)(\sin x+3)}{\sin x+3}=-\sin x \Rightarrow 3g'(x)-f'(x)=-\cos x \xrightarrow{x=\frac{\pi}{3}} -\cos\frac{\pi}{3}=-\frac{1}{2}$ ✓

Caline

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(۵)

$$g(x)=\frac{1}{x^5+|x^5|}\xrightarrow{\sqrt[5]{3}>0}\frac{1}{2x^5}\Rightarrow g(\sqrt[5]{3})=\frac{1}{6}$$

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$$f(x)=-\frac{1}{\sqrt[5]{x+|x|}}\xrightarrow{\frac{1}{6}>0} f(x)=\frac{-1}{\sqrt[5]{2x}}\Rightarrow f\circ g=\frac{-1}{\sqrt[5]{\frac{2}{2x^5}}}=-x$$

$$\Rightarrow (f\circ g)'(x)=-1$$

(۶)

$$f(x)=x\,g(x)+1\Rightarrow g(x)=\frac{f(x)-1}{x}\Rightarrow \lim_{x\to 0} g(x)=$$ تعریف مشتق در صفر است $=c$

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$$f(x)\left(\frac{-1+\sin x}{1+\sin x}\right)^2\Rightarrow f'(x)=2\left(\frac{\cos x(\sin x+1)-\cos x(\sin x-1)}{(1+\sin x)^2}\right)$$

$$\left(\frac{-1+\sin x}{1+\sin x}\right)\Rightarrow f'(0)=2\left(\frac{2\cos(0)}{(1+\sin(0))^2}\right)\left(\frac{\sin(0)-1}{1+\sin(0)}\right)=2\times2\times-1=$$ -4

خط d منحنی y_1 را در نقطه A و B قطع کند چون خط d موازی محور xها است ← نقاط A و B ← عرض یکسان، طول قرینه دارند.

(۷)

$$A(\alpha,\beta)\ ,\ B(-\alpha,\beta)\longrightarrow A(\tfrac{1}{2},\beta)\ ,\ B(-\tfrac{1}{2},\beta)$$

$$f(x)=-x^2-1\Rightarrow f'(x)=-2x\Rightarrow$$

$$mL_1=y_1'(-\alpha)=-2(-\alpha)=2\alpha$$

$$mL_2=y_1'(\alpha)=-2\alpha$$

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L_2, L_1 عمود اند $\longrightarrow mL_1\times mL_2=-1\longrightarrow -2\alpha\cdot2\alpha=-1\to 4\alpha^2=1\to\alpha=\pm\frac{1}{2}$

فاصله خط از مبدا مختصات $\to|\beta|\longrightarrow \beta=y_1(\frac{-1}{2})=-(-\frac{1}{2})^2-1=-\frac{1}{4}-1=\frac{-5}{4}=-1{,}25\longrightarrow |\beta|=1{,}25$

(۸)

آهنگ متوسط تغییر

$$f(x)=2\sqrt{x}(4x^2+3)\Rightarrow f'(x)=20x\sqrt{x}-\frac{3}{\sqrt{x}}\longrightarrow\frac{\Delta y}{\Delta x}=\frac{f(t)}{t}$$

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$$\Rightarrow\frac{2\sqrt{t}(4t^2+3)}{t}=20x^{\frac{3}{2}}+3\sqrt{t}\Rightarrow 8t^{\frac{5}{2}}+6t^{\frac{1}{2}}=20t^{\frac{5}{2}}+3t^{\frac{1}{2}}$$

$$3t^{\frac{1}{2}}=12t^{\frac{5}{2}}\Rightarrow 4t^2=1\Rightarrow t=\frac{1}{2}\Rightarrow f'(\tfrac{1}{2})=20\times\frac{1}{2\sqrt{2}}+3\sqrt{2}=8\sqrt{2}$$

$$4\left(\frac{1}{4\sqrt{x}}\right)(4x^2+3) + 2\sqrt{x}\,(8x^2+3)$$

$$\frac{4x^2+3}{\sqrt{x}} + \frac{16x\sqrt{x}+4\sqrt{x}}{1} = \frac{\qquad}{\sqrt{x}}$$

$$f(x) = \frac{\sqrt{x}}{-2x^2+x+1} \Rightarrow f'(x) = \frac{\left(\frac{1}{2\sqrt{x}}\right)(-2x^2+x-1) - (-4x+1)\sqrt{x}}{(-2x^2+x+1)^2}$$ (9) (4)

$$= \frac{6x^2-x+1}{2\sqrt{x}(-2x^2+x+1)} = \frac{\sqrt{x}}{2(-2x^2+x+1)x\sqrt{x}} \Rightarrow 6x^2-x+1 = -4x^2+2x+2$$ ✓

$$\Rightarrow 10x^2-3x-1 = 0 \xrightarrow{\text{دو ریشه}} x=\frac{1}{2}\ ✓ \quad x=-\frac{1}{5}\ ✗ \Rightarrow f\left(\frac{1}{2}\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$f(x) = (x[3])^3,\quad g(x) = \frac{1}{\sqrt{x^2-1}},\quad \lim_{x\to\frac{\sqrt{5}}{2}} g(x) = \frac{1}{\frac{1}{2}} = 2^+$$ (3) (10)

$$\Rightarrow fog'(\ ) = \frac{-24x}{\sqrt{(x^2-1)^5}} = fog'\left(\frac{\sqrt{5}}{2}\right) = \frac{-12\sqrt{5}}{\frac{1}{32}} = \frac{-12\sqrt{5}\times 32}{-48\sqrt{5}}$$ (8) ✓