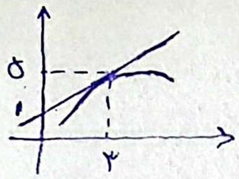
آرمین نوری — ۱۴,۵

۱)



$m = \frac{\Delta y}{\Delta x} = \frac{5-1}{3} = \frac{4}{3}$

۲

---

۲)

$\frac{a}{2\sqrt{ax-1}} = \frac{1}{3} \rightarrow 3a = 2\sqrt{ax-1} \rightarrow 9a^2 = 4(ax-1)\ ,\ \sqrt{ax-1} = \frac{3a}{2}$

$y = \frac{1}{3}x + \frac{4}{3} = \sqrt{ax-1} \rightarrow x = \frac{9a-8}{2}$ ($\sqrt{ax-1} \to \frac{3a}{2}$)

$\rightarrow 9a^2 = 18a^2 - 16a - 4$

$9a^2 - 16a - 4 = 0 \rightarrow a = 2 \checkmark \ ,\ a = -\frac{2}{9} \times$

$f(5) = \sqrt{9} = 3$

۲

---

۳)

$y = \frac{3}{4}x + \frac{n}{4} \xrightarrow{x=1} y = \frac{3+n}{4} \rightarrow \left|\begin{matrix} 1 \\ \frac{3+n}{4} \end{matrix}\right.$

$y = \frac{x^2+mx+1}{x+3} \xrightarrow{x=1} y = \frac{m+2}{4} \rightarrow \left|\begin{matrix} 1 \\ \frac{m+2}{4} \end{matrix}\right.$

$\rightarrow m+2 = 3+n \quad m-n = 1 \rightarrow n = 1$

۲

$\frac{(2x+m)(x+3) - (x^2+mx+1)}{(x+3)^2} = \frac{3}{4} \rightarrow \frac{x^2+6x+3m-1}{(x+3)^2} = \frac{3}{4}$

$\frac{6+3m}{16} = \frac{3}{4} \rightarrow 2+m = 4 \rightarrow m = 2$

$m+n = 3$

---

۴)

$3g(x) - f(x) = \frac{9}{3+\sin x} - \frac{(3-\sin x)(9+\sin^2 x+3\sin x)}{(3-\sin x)(3+\sin x)} = \frac{9-9-\sin^2 x-3\sin x}{3+\sin x}$

$= \frac{-\sin^2 x - 3\sin x}{3+\sin x} = \frac{-\sin x(\sin x+3)}{3+\sin x} = -\sin x \xrightarrow{\text{مشتق}} -\cos x \xrightarrow{x=5\pi/3} -\cos\frac{5\pi}{3} = -\frac{1}{2}$

۲

---

۵)

$g(x) = \frac{1}{2x^5} \qquad f(x) = \frac{-1}{\sqrt[5]{2x}}$

$(f(g(x)))' = g'(x) \times f'(g(x))$

$f(g(x)) = \frac{-1}{\sqrt[5]{2x \cdot \frac{1}{2x^5}}} = \frac{-1}{\sqrt[5]{\frac{1}{x^5}}} = -x \xrightarrow{\text{مشتق}} -1$

۲

آرمین نوری

۶)

$g(x)=\frac{f(x)-1}{x} \rightarrow \lim_{x\to 0} \frac{f(x)-1}{x}=\frac{0}{0}$

$\xrightarrow{Hop} 2\left(\frac{\sin x-1}{\sin x+1}\right)\left(\frac{\cos x(\sin x+1)-\cos x(\sin x-1)}{(\sin x+1)^2}\right)$ ✓

۲

$=-2\left(\frac{2\cos x}{(\sin x+1)^2}\right)=\frac{-4}{1}=-4$

---

۷)

$y=x^2+1 \xrightarrow{\text{قرینه نسبت به } x} y=-x^2-1 \quad y'=-2x$ $\quad \begin{vmatrix}\alpha\\ -\alpha^2-1\end{vmatrix} \quad \begin{vmatrix}-\alpha\\ -\alpha^2-1\end{vmatrix}$

$-2\alpha \cdot 2\alpha=-4\alpha^2=-1 \quad \alpha^2=\frac{1}{4} \quad \alpha=\pm\frac{1}{2}$ $\quad \begin{vmatrix}1/2\\ -5/4\end{vmatrix} \quad \begin{vmatrix}-1/2\\ -5/4\end{vmatrix}$

فاصله $=\frac{5}{4}$ ✓

۲

---

۸)

$y=ax \rightarrow$ خط $\quad 2\sqrt{B}(4B^2+3)=aB \quad ①$

۱/۵

$\begin{vmatrix}B\\ aB\end{vmatrix} \quad \frac{1}{\sqrt{x}}(4x^2+3)+8x(2\sqrt{x})=\frac{4x^2+3}{\sqrt{x}}+16x\sqrt{x}=a$

$\frac{4B^2+3}{\sqrt{B}}+16B\sqrt{B}=a \quad \frac{aB}{2\sqrt{B}\times\sqrt{B}}+16B\sqrt{B}=a \quad 16B\sqrt{B}=a/2$

$a=32B\sqrt{B} \quad \cancel{2\sqrt{B}}(4B^2+3)=\cancel{32}B^2\cancel{\sqrt{B}}$ (16) $\quad 4B^2+3=16B^2$

$12B^2=3 \quad B^2=\frac{1}{4} \quad B=\frac{1}{2} \quad a=\frac{16}{\sqrt{2}}$

$m=\frac{20(\frac{1}{4})+3}{\sqrt{\frac{1}{2}}}=8\sqrt{2}$

---

۹)

$f'(x)=\frac{\frac{6x^2-x+1}{2\sqrt{x}}}{((x+1/2)(x-1))^2}=\frac{6x^2-x+1}{2\sqrt{x}((x+1/2)(x-1))^2} \quad y=ax \quad \frac{\sqrt{x}}{-2x^2+x+1}=ax$

۲

$\frac{\sqrt{x}}{-2x^2+x+1}=x\times\frac{6x^2-x+1}{2\sqrt{x}((x+1/2)(x-1))^2} \rightarrow 6x^2-x+1=-4x^2+2x+2 \quad 10x^2-3x-1=0 \quad x\begin{cases}1/2 ✓\\ -1/5\end{cases}$ ✓

$A(1/2, \sqrt{2}/2) \quad$ عرض $=\sqrt{2}/2$

غیر قابل قبول

---

۱۰)

$f\circ g \rightarrow \left(\frac{1}{\sqrt{x^2-1}}\left[\frac{1}{1/10}\right]\right)^3=\left(\frac{10}{\sqrt{x^2-1}}\right)^3 \rightarrow 3\left(\frac{2}{\sqrt{x^2-1}}\right)^2\left(-2x(x^2-1)^{-3/2}\right)$

$\frac{\cancel{3}(\cancel{16})(-8\sqrt{5})}{-48\sqrt{5}}=8$ ✓

۲