محمدرضا دعوریان / دوازدهم - سیر ۸

۱-

$(0,1)$, $(3,5)$ $\Big|$ $m = \frac{5-1}{3-0} = 4/3$

$\Rightarrow y = 4/3\,x + 1 \Rightarrow y' = \frac{4}{3} = f'(x)$

$\Rightarrow \boxed{f'(3) = 4/3}$

۲-

شیب خط $m = \frac{2-1}{2-(-1)} = 1/3 \Rightarrow y = \frac{1}{3}x + 4/3$

$\Rightarrow \frac{1}{3}x + \frac{4}{3} = \sqrt{ax-1} \Rightarrow \frac{(4+x)^2}{9} = ax - 1$

$\Rightarrow (x+4)^2 = 9ax - 9$

$\Rightarrow x^2 - (8-9a)x + 25 = 0 \Rightarrow (x-5)^2$

$\Rightarrow 8 - 9a = \pm 10 \Rightarrow a = -2/9, 2$

$\Rightarrow f(5) = \sqrt{2(5) - 1} = \boxed{3}$

۳-

$4y - 3x = n \Rightarrow m = -\frac{(-3)}{4} = 3/4$

$y = (1, \frac{2+m}{4}) \Rightarrow y' = \frac{(2+m)(4) - (2+m)}{(4)^2} = \frac{3}{4}$

$\Rightarrow \boxed{m = 2}$

$y = \frac{3}{4}x + n/4 \Rightarrow 3/4 + n/4 = \frac{2+m}{4} \Rightarrow \boxed{n = 1}$

$\Rightarrow m + n = 2 + 1 = \boxed{3}$

۴-

$(3g - f)(x) = \frac{9}{3+\ln x} - \frac{(3-\ln)(9+\ln^2+3\ln)}{(3-\ln)(3+\ln)}$

$\Rightarrow \frac{9 - 9 - \ln^2 x - 3\ln x}{3 + \ln x} \Rightarrow \frac{-\ln x(3+\ln x)}{3+\ln x}$

$= -\ln x \Rightarrow (3g - f)'(x) = -\ln x$

$\Rightarrow -\ln(\frac{5\pi}{3}) = \boxed{-1/2}$

۵-

$(f\circ g)'(\sqrt[5]{3})$

$f(g(x)) = f(\frac{+1}{2x^5}) \Rightarrow \frac{-1}{\sqrt[5]{\frac{1}{x^5}}} = x$

$\Rightarrow (f\circ g)'(x) = (x)' = \boxed{1}$

۶-

$\frac{1+\ln^2 x - 2\ln x}{1+\ln^2 x + 2\ln x} = x g(x) + 1$

$\Rightarrow g(x) = \frac{-4\ln x}{(x)(\ln x + 1)^2} \Rightarrow \lim_{x\to 0} g(x) \xrightarrow{0/0}$

$\Rightarrow \text{Hop} = \frac{-4\ln x}{(1)(\ln x+1)^2 - (x)(2(\ln x+1)\ln x)}$

$\Rightarrow \frac{-4\ln(0)}{(\ln(0)+1)^2} = \boxed{-4}$

۷-

قرینه‌سری $\Rightarrow f(x) = -(x^2+1) \Rightarrow f'(x) = -2x$

$f'(\alpha)f(\alpha) = -1 \Rightarrow (-2\alpha)(2\alpha) = -1 \Rightarrow \alpha^2 = 1/4$

$\xrightarrow{\alpha>0} \alpha = 1/2 \Rightarrow f(1/2) = -((\frac{1}{2})^2+1) = -\frac{5}{4}$

$\Rightarrow (+1/2, -5/4) \Rightarrow y = -\frac{5}{4} \Rightarrow$ $\boxed{\text{فاصله از مبدأ } \frac{5}{4}}$

خط یک تابع ثابته

۸-

$f(x) = 8x^2\sqrt{x} + 6\sqrt{x} \rightarrow f'(x) = 20x\sqrt{x} + \frac{3}{\sqrt{x}}$

نقطه $(\alpha, d\alpha)$ را محل تماس می‌گیریم →

$f'(\alpha) = 20\alpha\sqrt{\alpha} + \frac{3}{\sqrt{\alpha}} = d \Rightarrow 20\alpha^2\sqrt{\alpha} + 3\sqrt{\alpha} = d\alpha$

$d\alpha = 20\alpha^2\sqrt{\alpha} + 3\sqrt{\alpha} = 8\alpha^2\sqrt{\alpha} + 6\sqrt{\alpha}$

$\Rightarrow 12\alpha^2\sqrt{\alpha} = 3\sqrt{\alpha} \rightarrow \alpha^2 = 1/4 \rightarrow \alpha = \pm 1/2$ (غ ق ق)

$f'(1/2) = 8(\frac{1}{2})^2\sqrt{1/2} + 6\sqrt{1/2} = \frac{d}{2} \Rightarrow \frac{8}{\sqrt{2}} = \frac{d}{2}$

$\Rightarrow d = \boxed{8\sqrt{2}}$

-۹.

$d: y = cx$

$f(\alpha) = \dfrac{\sqrt{\alpha}}{-2\alpha^2+\alpha+1} = c\alpha$

$\Rightarrow \dfrac{1}{-2\alpha^2+\alpha+1} = c\sqrt{\alpha} \rightsquigarrow c\sqrt{\alpha}(-2\alpha^2+\alpha+1) = 1$

$\Rightarrow -2c\alpha^2\sqrt{\alpha} + c\sqrt{\alpha}\,\alpha + c\sqrt{\alpha} = 1$

$\overset{'}{\Rightarrow} -5\alpha\sqrt{\alpha} + \dfrac{3}{2}\sqrt{\alpha} + \dfrac{1}{2\sqrt{\alpha}} = 0$

$\Rightarrow -10\alpha^2 + 3\alpha + 1 = 0 \to \alpha = -1/5 \,,\, \boxed{1/2}$

$f(\alpha) = \dfrac{\sqrt{1/2}}{(-2)(\frac{1}{2})^2 + (\frac{1}{2}) + 1} = \boxed{\sqrt{2}/2}$

-۱۰

$(f\circ g)'(x) = g'(x)\, f'(g(x))$

$g(x) = (x^2-1)^{-1/2} \xrightarrow{\text{مشتق}} \dfrac{-1}{2}(x^2-1)\,2x = -x(x^2-1)$

$f(x) = \dfrac{(x(x-1))^3}{x^2-x} \xrightarrow{\text{مشتق}} 3(x^2-x)(2x-1)$

$\rightsquigarrow g'\left(\dfrac{\sqrt{5}}{2}\right) = -\left(\dfrac{\sqrt{5}}{2}\right)\left(\dfrac{1}{4}\right) = \dfrac{-\sqrt{5}}{8}$

$f'(g(x)) = 3\left(\dfrac{1}{x^2-1} - \dfrac{1}{\sqrt{x^2-1}}\right)\left(\dfrac{2}{\sqrt{x^2-1}} - 1\right)$

$\Rightarrow (3)\left(\dfrac{1}{\frac{1}{4}} - \dfrac{1}{\frac{1}{2}}\right)\left(\dfrac{2}{\frac{1}{2}} - 1\right)$

$\Rightarrow (3)(2)(3) = 18$

$\rightsquigarrow (f\circ g)'(x) = 18 \times \dfrac{(-\sqrt{5})}{8} = \dfrac{-9\sqrt{5}}{4}$

$\rightsquigarrow \dfrac{+9\sqrt{5}}{4} \times \dfrac{1}{+49\sqrt{5}} = \boxed{\dfrac{3\sqrt{5}}{320}}$