

$$\begin{aligned} f(3) &= 5 \\ f(0) &= 1 \end{aligned} \Rightarrow m = \frac{5-1}{3-0} = \frac{4}{3} \Rightarrow f'(3) = \frac{4}{3}$$

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$$\begin{aligned} f(-1) &= 1 \\ f(2) &= 2 \end{aligned} \Rightarrow m = \frac{2-1}{2-(-1)} = \frac{1}{3} \Rightarrow y-1 = \frac{1}{3}(x+1) \Rightarrow y = \frac{1}{3}x + \frac{4}{3}$$

$$\Rightarrow \sqrt{ax-1} = \frac{1}{3}x + \frac{4}{3} \Rightarrow ax-1 = \frac{(x+4)^2}{9} \quad (1) \quad , \quad f'(2) = \frac{a}{2\sqrt{ax-1}} = \frac{1}{3} \Rightarrow 9a^2 = 4(ax-1)$$

$$\Rightarrow \frac{ax-1}{\frac{1}{3}} = \frac{9a^2}{4} \quad (2) \Rightarrow \frac{(x+4)^2}{9} = \frac{9a^2}{4} \Rightarrow x+4 = \frac{9a}{2} \Rightarrow x = \frac{9a}{2} - 4$$

$$\xrightarrow{(1)} a\left(\frac{9a}{2} - 4\right) - 1 = \frac{9a^2}{4} \Rightarrow a = 2 \Rightarrow f(5) = \sqrt{5(2)-1} = 3$$

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$$y = \frac{x^2 + mx + 1}{x+3} \Rightarrow y' = \frac{12x^2 + 4x + 3m - 1}{(x+3)^2} \Rightarrow f'(1) = \frac{4+3m}{14}$$

$$4y - 3x = n \Rightarrow y = \frac{3}{4}x + \frac{n}{4} \Rightarrow y' = \frac{3}{4}$$

$$\Rightarrow \frac{4+3m}{14} = \frac{3}{4} \Rightarrow m = 2$$

$$y = \frac{1+x+1}{1+x} = 1 \Rightarrow A(1,1) \Rightarrow 4y - 3x = n \Rightarrow 4(1) - 3(1) = n \Rightarrow n = 1$$

$$\Rightarrow m+n = 2+1 = 3$$

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$$\begin{aligned} & 3g'\left(\frac{\sin x}{3}\right) - f'\left(\frac{\sin x}{3}\right) = (3g - f)' = \left(\frac{9}{3+\sin x} - \frac{2\sqrt{1-\sin^2 x}}{1-\sin^2 x}\right)' \\ & = \left(\frac{9}{3+\sin x} - \frac{(1-\sin x)(1+\sin x + \sin x)}{(1-\sin x)(1+\sin x)}\right)' = \left(\frac{9-1-3\sin x - \sin^2 x}{3+\sin x}\right)' = \left(\frac{-(\sin x)(\sin x+3)}{\sin x+3}\right)' \\ & = -(\sin x)' = -\cos x \xrightarrow{x=\frac{\pi}{3}} -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2} \end{aligned}$$

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$$g'(f(x)) \cdot f'(g(x)) = (f \circ g)'(x)$$

$$f(g(x)) = f\left(\frac{1}{x^2}\right) = -\frac{1}{\sqrt[5]{2\left(\frac{1}{x^2}\right)}} = -\frac{1}{x^{-1}} = -x \Rightarrow (f \circ g)'(x) = (-x)' = -1$$

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$$f(x) = \left(\frac{\sin x - 1}{\sin x + 1} \right)^r \rightarrow f'(x) = r \left(\frac{\sin x - 1}{\sin x + 1} \right) \left(\frac{r \cos x}{(\sin x + 1)^r} \right) \rightarrow f'(0) = r(-1)(r) = -r$$

$$f(x) = xg(x) + 1 \rightarrow g(x) = \frac{f(x) - 1}{x}$$

$$\lim_{x \rightarrow 0} g(x) = \lim_{x \rightarrow 0} \frac{f(x) - 1}{x} = f'(0) \rightarrow \lim_{x \rightarrow 0} g(x) = f'(0) = -r$$

$$y = x^2 + 1 \xrightarrow{\text{قیم بستن به محور } x} y = -x^2 - 1 \xrightarrow{y=k} -x^2 - 1 = k \rightarrow x^2 = -1 - k \rightarrow x = \pm \sqrt{-1 - k}, k < -1$$

$$y' = -2x \rightarrow \begin{cases} m_1 = -2\sqrt{-1-k}, x_1 = +\sqrt{-1-k} \\ m_2 = 2\sqrt{-1-k}, x_2 = -\sqrt{-1-k} \end{cases} \xrightarrow{\text{مورد ۳م}} m_1 \times m_2 = -1$$

$$\rightarrow -r(-1-k) = -1 \rightarrow 1+k = \frac{-1}{r} \Rightarrow k = \frac{-\frac{1}{r}}{r} \quad \text{فاصله خط از مبدأ مختصات} = |K| = \frac{\frac{1}{r}}{r}$$

$$f(x) = r(x)^{\frac{1}{r}}(rx^r + r) \rightarrow f'(x) = r \left[\frac{1}{r} x^{-\frac{1}{r}} (rx^r + r) + x^{\frac{1}{r}} \cdot 1 \right] = r \left[\frac{rx^r + r}{r x^{\frac{1}{r}}} + 1 \right]$$

$$= \frac{rx^r + r}{x^{\frac{1}{r}}} + 14x^{\frac{r}{r}} \quad , \quad f(a) = ma \rightarrow r \sqrt[r]{a} (ra^r + r) = ma \rightarrow m = \frac{r \sqrt[r]{a} (ra^r + r)}{a} = \frac{r (ra^r + r)}{\sqrt[r]{a}} \quad (I)$$

$$f'(a) = m \rightarrow \frac{ra^r + r}{\sqrt[r]{a}} + 14a^{\frac{r}{r}} = m \xrightarrow{(I)} \frac{ra^r + r}{\sqrt[r]{a}} + 14a^{\frac{r}{r}} = \frac{r (ra^r + r)}{\sqrt[r]{a}} \rightarrow 14a^{\frac{r}{r}} = \frac{ra^r + r}{\sqrt[r]{a}}$$

$$\rightarrow 14a^r = ra^r + r \rightarrow a = \frac{1}{r} \rightarrow m = \frac{r (ra^r + r)}{\sqrt[r]{a}} = \frac{r (r \cdot \frac{1}{r^r} + r)}{\sqrt[r]{\frac{1}{r}}} = \frac{1}{\frac{1}{r}} = 1/r$$

$$f(x) = \frac{\sqrt{x}}{-2x^2x+1} \quad , \quad y = mx \rightarrow f(A) = mA \rightarrow m = \frac{f(A)}{A} = f'(A)$$

$$\frac{f(x)}{x} = \frac{\sqrt{x}}{x(-2x^2x+1)} = \frac{1}{\sqrt{x}(-2x^2x+1)} \rightarrow f'(x) = \frac{4x^r - 2 + 1}{2\sqrt{x}(-2x^2x+1)^r} \xrightarrow{f(A) = \frac{f(A)}{A}} \frac{4A^r - A + 1}{2\sqrt{A}(-2A^2A+1)^r}$$

$$\rightarrow \left. \begin{matrix} A = \frac{1}{r} \\ A = \frac{1}{\omega} \end{matrix} \right\} \xrightarrow{x \rightarrow 0} A = \frac{1}{r} \rightarrow f\left(\frac{1}{r}\right) = \frac{\sqrt{\frac{1}{r}}}{-2\left(\frac{1}{r}\right)^r + \frac{1}{r} + 1} = \frac{1}{r} = \frac{\sqrt{r}}{r} \quad = \frac{1}{\sqrt{A}(-2A^2A+1)^r}$$

$$g\left(\frac{\sqrt{10}}{r}\right) = \frac{1}{\sqrt{\frac{10}{r}} - 1} = \frac{1}{\sqrt{\frac{1}{r}}} = r \rightarrow \log\left(\frac{\sqrt{10}}{r}\right) = f(r)$$

$$x \rightarrow r \Rightarrow [x] = 1 \rightarrow f(x) = x^r \rightarrow f'(r) = r x^{r-1} = r(r)^{r-1} = 1r$$

$$g(x) = (x^r - 1)^{\frac{1}{r}} \rightarrow g'(x) = \frac{-x}{(x^r - 1)^{\frac{r-1}{r}}} \rightarrow g'\left(\frac{\sqrt{10}}{r}\right) = -r\sqrt{10}$$

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x) \rightarrow (f \circ g)'\left(\frac{\sqrt{10}}{r}\right) = 1r \times (-r\sqrt{10}) = -r^2\sqrt{10} \rightarrow \frac{(f \circ g)'\left(\frac{\sqrt{10}}{r}\right)}{-r^2\sqrt{10}} = 1$$