نام و نام خانوادگی ......عرفان عبداللهی...... ۱۷٫۵ پاسخنامه تشریحی تکلیف شماره ...۲۳... کلاس ......دوازدهم سر A

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$$g'(x) = f'(\tan^2 x + \sqrt{2}\cos x) \times (2\times \tan x(1+\tan^2 x) - \sqrt{2}\sin x)$$

$$g'(\frac{\pi}{4}) = f'(1 + \frac{\sqrt{2}}{\sqrt{2}}) \times (2\times 1\times(1+1) - \frac{\sqrt{2}}{\sqrt{2}})$$

$$\sqrt{3} = f'(2)\times(3) \longrightarrow f'(2) = \frac{\sqrt{3}}{3}$$

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$$f(x) = \frac{(2+\cos x)(\cos^2 x + 4 - 2\cos x)}{(2-\cos x)(2+\cos x)} = \frac{\cos^2 x + 4 - 2\cos x}{2-\cos x}$$

$$f(x) = (2-\cos x) + \frac{+2\cos x}{2-\cos x} \rightarrow f(x) = (2-\cos x) + \cos x \; g(x)$$

$$f'(x) = +\sin x + \cos(x)\, g'(x) - \sin(x)\, g(x) \rightarrow f'(\frac{7\pi}{6}) = -\frac{1}{2}\left(1-\frac{\sqrt{3}}{2}\right) g'(\frac{7\pi}{6})$$

$$f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6}) = -\frac{1}{2} - \left(\frac{3+\sqrt{3}}{2}\right) g'(\frac{\pi}{6}) = -\frac{1}{2} - \left(\frac{3+\sqrt{3}}{2}\right)\left(\frac{-1}{(2-\frac{\sqrt{3}}{2})^2}\right)$$

(جواب پایین صفحه)

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$$f(x) = |4x-3|\sqrt{ax}$$

$$f'_+(\frac{3}{4}) = (4x-3)'\sqrt{ax} \longrightarrow 4\sqrt{\frac{3}{4}a} \longrightarrow 8\sqrt{\frac{3}{4}a} = 2\sqrt{6}$$

$$f'_-(\frac{3}{4}) = -(4x-3)'\sqrt{ax} \longrightarrow -4\sqrt{\frac{3}{4}a} \qquad 4\sqrt{\frac{3}{4}a} = \sqrt{6}$$

$$\sqrt{16\times\frac{3}{4}a} = \sqrt{6} \rightarrow 12a = 6 \qquad \boxed{a = \frac{1}{2}}$$

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$$y = -ax + 2 \qquad f'(4) = -a \qquad f(4) = -4a + 2$$

$$f(4) + f'(4) = -5a + 2 = -1 \qquad 5a = 3$$

$$a = \frac{3}{5} \qquad f'(4) = -\frac{3}{5}$$

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$$y = \frac{x}{7} + \frac{5}{7} \qquad f'(x) = \frac{1}{7} \qquad f(x) = \frac{x}{7} + \frac{5}{7}$$

$$f(x) = \frac{ax-1}{3x+1} \qquad \frac{a+3}{(3x+1)^2} = \frac{1}{7} \qquad \frac{7(ax-1)}{3x+1} = x+5$$

$$f'(x) = \frac{a+3}{(3x+1)^2} \qquad a = \frac{(3x+1)^2}{7} - 3 \qquad 7ax - 7 = 3x^2 + 16x + 5$$

$$x = 2 \;,\; a = 4$$

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$f(2)=0 \qquad f'(2)=0 \qquad b=-16 \qquad a=-12$ $f'(x)=3x^2+a \longrightarrow a=-12$ $f(x)=x^3+ax-b \longrightarrow 8-24-b=0 \longrightarrow b=-16$ $b-a=-16+12=-4$ ✓ ۲	۶
$\lim\limits_{x\to 0}\dfrac{f(x)\ f'(x)}{(1-\cos x)^m}=32\sqrt{2} \rightarrow f'(x)=n\sin^{n-1}(x^2)\cos(x^2).2x$ $\longrightarrow \dfrac{x^{2n}\times n\times x^{2n-2}\times(1-\frac{x^4}{2})\times 2x}{(1-(1-\frac{x^2}{2}))^m} \qquad m=2n-\frac{1}{2}$ $\sin(x^2)\sim x^2$ $\cos(x^2)\sim 1-\frac{x^4}{2}$ $f(x)=\sin^n(x^2)\sim x^{2n}$ ✓ $= \dfrac{2n\,x^{4n-1}(1-\frac{x^4}{2})\times 2^m}{x^{2m}} \Rightarrow 2n2^m=2^{\frac{11}{2}}$ $2m+n=2(\frac{7}{2})+2=9 \qquad n=2 \qquad m=\frac{7}{2}$ ۲	۷
$f(x)=\dfrac{\sqrt{x}+1}{\sqrt{x}-1} \rightarrow f'(x)=\dfrac{-2}{(\sqrt{x}-1)^2} \qquad f(9)=2 \quad f^{-1}(2)=9$ ۱ $f(x)=\dfrac{u+1}{u-1} \Rightarrow f'(x)=\dfrac{-2}{(u-1)^2} \qquad f'(9)=\dfrac{-2}{4}=-\dfrac{1}{2}$ $f^{-1'}(2)=-2$ (جواب پایین صفحه)	۸
$\dfrac{f(0)-f(-1)}{0-(-1)}=\dfrac{1-(2)^3(1-a)}{1}=-11 \rightarrow -11=1-8+8a$ $-4=8a$ $-\frac{1}{2}=a$ $x=-2(-\frac{1}{2})=1$ $f'(1)=3\times 2x(x^2+1)^2(1-\frac{1}{2}x)-\frac{1}{2}(x^2+1)^3=12-4=8$ ✓ ۲	۹
$y=\sin x\ \cos 2x \rightarrow$ $y=(\sin^2+\cos^2)(\sin^2-\cos^2)=\sin^2-\cos^2=-\cos 2x$ $\dfrac{(\sin(\frac{\pi}{2})\cos(\pi))-(\sin(\frac{\pi}{2})\cos\frac{\pi}{2})}{4\pi}\times\dfrac{4\pi}{-\cos\pi+\cos\frac{\pi}{2}}=\dfrac{1}{-1}$ ✓ ۲	۱۰ -1

سوال ۱۲

$$f'\left(\frac{7\pi}{6}\right)-2g'\left(\frac{7\pi}{6}\right)=(f-2g)'\left(\frac{7\pi}{6}\right)$$

$$\to (f-2g)'=\left(\frac{8+\cos^3 x}{4-\cos^2 x}-\frac{4}{2-\cos x}\right)=\left(\frac{(\cos x+2)(\cos^2 x-2\cos x+4)}{(2-\cos x)(2+\cos x)}-\frac{4}{2-\cos x}\right)=$$

$$\left(\frac{\cos(\cos x-2)}{-(\cos x-2)}\right)'=-(\cos x)'=\sin x \xrightarrow{x=7\pi/6} \sin\left(\frac{7\pi}{6}\right)=\frac{-1}{2}$$

سوال ۱۹

$$f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1}\to \sqrt{x}y-y=\sqrt{x}+1\longrightarrow \sqrt{x}(y-1)=y+1\to \sqrt{x}=\frac{y+1}{y-1}\to x=\left(\frac{y+1}{y-1}\right)^2\longrightarrow y=\left(\frac{x+1}{x-1}\right)^2$$

$$f^{-1}(x)=\left(\frac{x+1}{x-1}\right)^2\to (f^{-1})'=2\left(\frac{x+1}{x-1}\right)\times\frac{-2}{(x-1)^2}\longrightarrow (f^{-1}(2))'=2\times3\times(-2)=-12$$