A دوازدهم تجربی — تکلیف شماره ۲۳ — ۲۰ — ایلیا سالاری

① $g(x)=f(\tan^2 x+\sqrt{2}\cos x)$, $g'(\frac{\pi}{4})=\sqrt{3}$ $\xrightarrow{\text{مشتق می‌گیریم}}$ $g'(x)=2((1+\tan^2 x)\tan x-\sqrt{2}\sin x)f'(\tan x+\sqrt{2}\cos x)\Rightarrow$

$\Rightarrow g'(\frac{\pi}{4})=(2(2)(1)-\frac{\sqrt{2}\times\sqrt{2}}{2})f'(1+\frac{\sqrt{2}\times\sqrt{2}}{2})\Rightarrow\sqrt{3}=3f'(2)\Rightarrow f'(2)=\frac{\sqrt{3}}{3}$ ✓ ۲

② $f(x)=\frac{8+\cos^3 x}{4-\cos^2 x}$ $\quad g(x)=\frac{2}{2-\cos x}$ $\quad g'(x)=\frac{-2\sin x}{(2-\cos x)^2}\Rightarrow g'(\frac{7\pi}{6})=\frac{4}{19+8\sqrt{3}}$

$f'(x)=\frac{(-3\sin x\cos^2 x)(4-\cos^2 x)-(2\sin x\cos x)(8+\cos^3 x)}{(4-\cos^2 x)^2}\Rightarrow f'(\frac{7\pi}{6})=\frac{(\frac{9}{8}\times\frac{13}{4})-(4\sqrt{3}-\frac{9}{16})}{\frac{169}{16}}$ ۲

$f'(\frac{7\pi}{6})-2g'(\frac{7\pi}{6})=\frac{135-128\sqrt{3}}{338}-\frac{8}{19+8\sqrt{3}}=\frac{-169}{338}=-\frac{1}{2}$ ✓

③ $f(x)=|4x-3|\sqrt{ax}\Rightarrow f'(x)\begin{cases}\lim_{x\to(\frac{3}{4})^+}\frac{(4x-3)\sqrt{ax}-0}{x-\frac{3}{4}}=4\sqrt{ax}=4\frac{\sqrt{3a}}{2}=2\sqrt{3a}\\ \lim_{x\to(\frac{3}{4})^-}\frac{-(4x-3)\sqrt{ax}-0}{x-\frac{3}{4}}=-4\sqrt{ax}=-4\frac{\sqrt{3a}}{2}=-2\sqrt{3a}\end{cases}$ ۲

$2\sqrt{3a}-(-2\sqrt{3a})=2\sqrt{6}\Rightarrow\sqrt{3a}=\frac{\sqrt{6}}{2}\Rightarrow 3a=\frac{6}{4}\Rightarrow a=\frac{1}{2}$ ✓

④ $f(4)+f'(4)=-1$ $\quad f'(4)=-a$, $f(4)=2-4a$ $\quad f(4)+f'(4)=-1\Rightarrow 2-4a-a=-1\Rightarrow a=\frac{3}{5}\Rightarrow f'(4)=-0.6$ ۲

⑤ $y=\frac{ax-1}{3x+1}$ $\quad 7y-x=5\Rightarrow y=\frac{x+5}{7}$ $\quad \frac{x+5}{7}=\frac{ax-1}{3x+1}\Rightarrow 3x^2+16x+5=7ax-7\Rightarrow$

$\Rightarrow 3x^2+(16-7a)x+12=0\Rightarrow\Delta=0\Rightarrow(16-7a)^2-4(12)(3)=0\Rightarrow\begin{cases}16-7a=12\Rightarrow 7a=4\Rightarrow a=\frac{4}{7} & \text{غ ق ق}\\ 16-7a=-12\Rightarrow 7a=28\Rightarrow a=4 & \text{ق ق}\end{cases}$ ✓ ۲

⑥ $f(x)=x^3+ax-b$ $\quad f(2)=0\Rightarrow 8+2a-b=0\Rightarrow b=-16$ $\quad f'(2)=0\Rightarrow 12+a=0\Rightarrow a=-12$ ۲

$(-16)-(-12)=-4$ ✓

⑦ $f(x)=\sin^n(x^2)$ $\quad \lim_{x\to 0}\frac{f(x)f'(x)}{(1-\cos x)^m}=32\sqrt{2}$ $\quad f'(x)=n\sin^{n-1}(x^2)\times\cos(x^2)\times 2x$

$\lim_{x\to 0}\frac{f(x)f'(x)}{(1-\cos x)^m}=\frac{2nx\cos(x^2)\sin^{2n-1}(x^2)}{(1-\cos x)^m}=\frac{(2nx)(1)(x^2)^{2n-1}}{\frac{x^{2m}}{2^m}}=\frac{(2n2^m)(x)(x^{4n-2})}{x^{2m}}=\frac{n\times 2^{m+1}\times x^{4n-1}}{x^{2m}}=2^{5.5}$ ۲

$2^{m+1}\times n=2^{5.5}\Rightarrow n=2$, $m=3.5$ $\quad 2m+n=7+2=9$ ✓

⑧ $f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1}\Rightarrow y=\frac{\sqrt{x}+1}{\sqrt{x}-1}\Rightarrow$ ۲

$\sqrt{x}y-y=\sqrt{x}+1\Rightarrow\sqrt{x}(y-1)=y+1\Rightarrow\sqrt{x}=\frac{y+1}{y-1}\Rightarrow f^{-1}(x)=(\frac{x+1}{x-1})^2$ $\quad (f^{-1})'(x)=2(\frac{x+1}{x-1})(\frac{(x-1)-(x+1)}{(x-1)^2})\Rightarrow$

$(f^{-1})'(2)=2\quad(-2)=-12$ ✓

⑨ $f(x)=(x^2+1)^3(ax+1)$ $\quad \frac{f(0)-f(-1)}{0-(-1)}=\frac{1-(8(1-a))}{1}=8a-7=-11\Rightarrow a=-\frac{1}{2}$ ۲

$f'(x)=3(2x)(x^2+1)^2(-\frac{x}{2}+1)+(-\frac{1}{2})(x^2+1)^3$ $\quad f'(1)=(3)(2)(4)(\frac{1}{2})-4=12-4=8$ ✓

⑩ $y_1=\sin x\cos 2x\xrightarrow{\text{آهنگ آن}}\frac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{-1-0}{\frac{\pi}{4}}=\frac{-4}{\pi}$ $\quad y_2=\sin^4 x-\cos^4 x\xrightarrow{\text{آهنگ}}\frac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=$

$=\frac{1-0}{\frac{\pi}{4}}=\frac{4}{\pi}$ $\quad \frac{y_1}{y_2}=\frac{\frac{-4}{\pi}}{\frac{4}{\pi}}=-1$ ✓ ۲