مقطع : دوازدهم پسر A   « به نام خدا »   نام و نام خانوادگی : حسام رفیع زاده

۱۷/۲۵

۱.

$$g(x) = f(\tan^2 x + \sqrt{2}\cos x) \xrightarrow{()'} g'(x) = f'(\tan^2 x + \sqrt{2}\cos x) \times (1+\tan^2 x + (-\sqrt{2}\sin x))$$

۱/۵

$$\xrightarrow{x=\frac{\pi}{4}} g'(\tfrac{\pi}{4}) = f'(1+1) \times (1+\cancel{1}-\cancel{1}) \Rightarrow f'(2) = g'(\tfrac{\pi}{4}) \to \boxed{f'(2) = \sqrt{3}}$$

$$g'(x) = (2\tan x(1+\tan^2 x) - \sqrt{2}\sin x) f'(\tan^2 x + \sqrt{2}\cos x)$$

$$\xrightarrow{x=\pi/4} g'(\tfrac{\pi}{4}) = 3f'(2) = \sqrt{3} \to f'(2) = \frac{1}{\sqrt{3}} \to f'(2) = \frac{\sqrt{3}}{3}$$

۲.

$$f(x) = \frac{8+\cos^3 x}{4-\cos^2 x} \to$$

$$\to f(x) - 2g(x) = \frac{\cos^3 x + 8 - 8 - 4\cos x}{4-\cos^2 x}$$

$$g(x) = \frac{2}{2-\cos x} = \frac{4+2\cos x}{4-\cos^2 x}$$

$$\to f(x) - 2g(x) = \frac{\cos^3 x - 4\cos x}{4-\cos^2 x} = \frac{\cos x(\cancel{\cos^2 x - 4})}{-\cancel{(\cos^2 x - 4)}} = -\cos x \xRightarrow{()'} \sin x$$

۲

$$\to f'(\tfrac{7\pi}{6}) - 2g'(\tfrac{7\pi}{6}) = \sin\tfrac{7\pi}{6} = \boxed{-\tfrac{1}{2}}$$ ✓

$$x \to (\tfrac{3}{4})^+ \to f(x) = (4x-3)\sqrt{ax} \to f'(x) = 4\sqrt{ax}$$ مشتق عامل صفر شونده $$\to f'_+(\tfrac{3}{4}) = 4\sqrt{\tfrac{3}{4}a}$$

$$x \to (\tfrac{3}{4})^- \to f(x) = (3-4x)\sqrt{ax} \to f'(x) = -4\sqrt{ax}$$ مشتق عامل صفر شونده $$\to f'_-(\tfrac{3}{4}) = -4\sqrt{\tfrac{3}{4}a}$$

$$f(x) = |4x-3|\sqrt{ax} \to f(x) = \begin{cases} (4x-3)\sqrt{ax} & x \geq \frac{3}{4} \\ (-4x+3)\sqrt{ax} & x < \frac{3}{4} \end{cases} \xrightarrow{()'} f'(x) = \begin{cases} 4\sqrt{ax} + \frac{a}{2\sqrt{ax}}(4x-3); & x \geq \frac{3}{4} \\ -4\sqrt{ax} + \frac{a}{2\sqrt{ax}} \times (-4x+3); & x < \frac{3}{4} \end{cases}$$

$$f'_+(\tfrac{3}{4}) = 2\sqrt{3a}$$

$$f'_-(\tfrac{3}{4}) = -2\sqrt{3a}$$

$$\xrightarrow{\text{اختلاف}} \cancel{4}\sqrt{3a} = \cancel{4}\sqrt{6} \to \boxed{a = 2}$$

۰

$$\to f'_+(\tfrac{3}{4}) - f'_-(\tfrac{3}{4}) = 8\sqrt{\tfrac{3}{4}a} = 2\sqrt{6} \to \sqrt{\tfrac{3}{4}a} = \frac{\sqrt{6}}{4} \to \tfrac{3}{4}a = \frac{6}{16} \to a = \frac{1}{2}$$

Scanned with CamScanner

۴-

$y+ax=1$, $f(x)$, $x=4$

$\xrightarrow[\text{شکل}]{\text{با توجه به}}$ $f'(4) = -a$, $f(4) = -4a+2$ $\xrightarrow{+}$ $-5a+2=-1 \rightarrow -5a=-3 \rightarrow a=\frac{3}{5}$

$\rightarrow f'(4) = \frac{-3}{5}$

2

۵-

باید محل برخورد $\Delta=0$ باشد ریشه مضاعف باشد.(1)

1,75

$f(x) = \frac{x+5}{7} \rightarrow \frac{ax-1}{3x+1} = \frac{x+5}{7} \rightarrow 3x^2+16x+5 = 7ax-7$

$\rightarrow 3x^2+16x+5-7ax+7=0 \xrightarrow{(1)} \Delta=0 \rightarrow (16-7a)^2-144=0 \rightarrow (16-7a)^2=144$

$16-7a=12 \rightarrow -7a=-4 \rightarrow a=\frac{4}{7}$ ×

$16-7a=-12 \rightarrow -7a=-28 \rightarrow a=4$

۶-

$f(x)=x^3+ax-b \xrightarrow{(1)'} f'(x)=3x^2+a \xrightarrow{x=2} 12+a=0 \rightarrow a=-12$

$f(2)=0 \xrightarrow{x=2} x^3-12x-b=0 \xrightarrow{x=2} 8-24-b=0 \rightarrow b=-16$

$b-a=-4$

2

۷-

$f(x)=\sin^n(x^2)$, $\lim_{x\to 0} \frac{f(x).f'(x)}{(1-\cos x)^m} = 32\sqrt{2}$, $f'(x) = n\sin^{n-1}(x^2) \times \cos(x^2) \times 2x$

$\lim_{x\to 0} \frac{f(x).f'(x)}{(1-\cos x)^m} \overset{\text{هم‌ارزی}}{\sim} \frac{2^m \times x^{2n} \times n \times x^{2n-2} \times 2x}{x^{2m}} \times \frac{\cos(x^2)}{1} = \frac{2^{m+1} \times \cancel{x^{4n-1}} \times n}{\cancel{x^{2m}}} = 32\sqrt{2}$

2

$2^{m+1} \times n = 32\sqrt{2}$

$4n-1=2m \rightarrow 2n-\frac{1}{2}=m$ (1)

$\xrightarrow{(1)} 2^{2n+1/2} \times n = 32\sqrt{2} \rightarrow n=2$, $m=\frac{7}{2}$ $\rightarrow 2m+n = 9$

۸-

$$f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1} \quad , \quad (f^{-1}(2))' = ? \quad / \quad f'(x)=\frac{\frac{1}{2\sqrt{x}}(\sqrt{x}-1)-\frac{1}{2\sqrt{x}}(\sqrt{x}+1)}{(\sqrt{x}-1)^2} \quad ①$$

$$f(x)=2 \rightarrow \frac{\sqrt{x}+1}{\sqrt{x}-1}=2 \rightarrow x=9 \ ,\overset{①}{\Rightarrow} f'(9)=\frac{-1}{12} \Rightarrow (f^{-1}(2))'=\frac{1}{f'(9)}$$

$$\rightarrow (f^{-1}(2))' = \boxed{-12}$$

۲

۹-

$$\text{آهنگ متوسط} = \frac{f(0)-f(-1)}{0-(-1)} \rightarrow \text{آهنگ متوسط} = 1+8a-8=-11 \rightarrow a=\frac{-1}{2}$$

$$\text{خواسته مسئله}: f'(1)=? \quad , \quad f'(x)=3(x^2+1)^2 \times 2x \times \left(\frac{-1}{2}x+1\right)+\frac{-1}{2}\times(x^2+1)^3$$

$$\rightarrow f'(1)=\boxed{8}$$

۲

۱۰-

$$f(x)=\sin x \cdot \cos 2x$$

$$g(x)=\sin^4 x-\cos^4 x$$

$$\text{آهنگ متوسط } f(x) = \frac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{4}} = \frac{-4}{\pi}$$

$$\text{آهنگ متوسط } g(x) = \frac{g(\frac{\pi}{2})-g(\frac{\pi}{4})}{\frac{\pi}{4}} = \frac{1-0}{\frac{\pi}{4}} = \frac{4}{\pi}$$

$$\rightarrow \frac{\text{آهنگ } f}{\text{آهنگ } g} = \boxed{-1}$$

۲

Scanned with CamScanner