۱۴٫۵

نام و نام خانوادگی ....طاها حق جویی.... پاسخنامه تشریحی تکلیف شماره ...۲۳... کلاس ...دوازدهم پسر A...

۱

$g'(x)=f'(\tan^2x+\sqrt{2}\cos x)\times(2\tan x.(1+\tan^2x)+\sqrt{2}(-\sin x))$

$\Rightarrow g'(\frac{\pi}{4})=f'(1^2+\sqrt{2}(\frac{\sqrt{2}}{2}))\times(2\times(1+1^2)+\sqrt{2}(-\frac{\sqrt{2}}{2}))=f'(2)\times 3=\sqrt{3}$

$\to f'(2)=\frac{\sqrt{3}}{3}$ ✓

۲

۲

$f'(x)=\frac{(3\cos^2x\times(-\sin x))(4-\cos^2x)-(-2\cos x(-\sin x))(8+\cos^3x)}{(4-\cos^2x)^2}$

۱٫۷۵

$\to f'(\frac{7\pi}{6})=\frac{135-128\sqrt{3}}{338}$ / $g'(x)=\frac{-2\sin x}{(2-\cos x)^2}\to g'(\frac{7\pi}{6})=\frac{76-32\sqrt{3}}{169}$

$\to f'(\frac{7\pi}{6})-g'(\frac{7\pi}{6})=-\frac{1}{2}$

$-(\cos x)'=\sin x\xrightarrow{x=\frac{7\pi}{6}}\sin(\frac{7\pi}{6})=\frac{-1}{2}$

۳

$f(x)=\begin{cases}(4x-3)\sqrt{ax} & x\geq\frac{3}{4}\\(-4x+3)\sqrt{ax} & x<\frac{3}{4}\end{cases}\to f'(x)=\begin{cases}4\sqrt{ax}+\frac{4x-3}{2\sqrt{ax}} & x>\frac{3}{4}\\-4\sqrt{ax}+\frac{-4x+3}{2\sqrt{ax}} & x<\frac{3}{4}\end{cases}$

۲

$\to f'_-(\frac{3}{4})=4\sqrt{\frac{3}{4}a}$, $f'_+(\frac{3}{4})=-4\sqrt{\frac{3}{4}a}\to f'_-(\frac{3}{4})-f'_+(\frac{3}{4})=8\sqrt{\frac{3}{4}a}=2\sqrt{6}$

$\to a=\frac{1}{2}$ ✓

۴

$\left.\begin{array}{l}x=4\to y=-4a+2\to f(4)=-4a+2\\ y'=-a\to f'(4)=-a\end{array}\right\}\xrightarrow{+}-5a+2=-1\to a=\frac{3}{5}$

۱٫۷۵

$f'(4)=-a=-\frac{3}{5}=-0.6$

۵

$y=\frac{1}{7}x+5\to\frac{7(ax-1)}{3x+1}=x+5\to 3x^2+(16-7a)x+12=0\xrightarrow{\Delta=0}$

۲

$(16-7a)^2-144\to 16-7a=\pm 12\to\begin{cases}a=\frac{4}{7}\to x=-2\ \times\\ a=4\to x=2,\ y=1\ ✓\end{cases}\to a=4$

در ناحیه اول

۶

$$(x-2)^2(dx+c) = \underset{d=1}{dx^3} + \underset{c=4}{cx^2} - 4dx^2 - 4cx + 4dx + 4c = x^3 + ax - b$$

$$\rightarrow x^3 - 12x + 16 = x^3 + ax - b \rightarrow a = -12 \,,\; b = -16 \rightarrow b - a = -16 + 12 = -4$$

۲

۷

$$\lim_{x\to 0} \frac{\sin^n(x^2) \times n\sin^{n-1}(x^2) \times \cos(x^2) \times 2x}{(1-\cos x)^m}$$

$$\lim_{x\to 0} \frac{\sin^n(x^2) \times n\sin^{n-1}(x^2) \times \cos(x^2) \times (2x)}{(2\sin^2\frac{x}{2})^m} \xrightarrow{\sin x \sim x} \lim_{x\to 0} \frac{(x^2)^n n (x^2)^{n-1} \times 2x}{(2(\frac{x}{2})^2)^m} =$$

$$\lim_{x\to 0} \frac{2nx^{2n+2n-2+1}}{(\frac{x^2}{2})^m} = \lim_{x\to 0} \frac{2nx^{4n-1}}{\frac{1}{2^m}x^{2m}} = \lim_{x\to 0} 2^{m+1} n x^{4n-2m-1}$$

حاصل حد عدد غیرمتناهی غیرصفر ← توان صفر باشد

$$\begin{cases} 4n - 2m - 1 = 0 \rightarrow n = \frac{2m+1}{4} \\ 2^{m+1} n = 32\sqrt{2} \end{cases} \longrightarrow 2^{m+1} \times \frac{2m+1}{4} = 32\sqrt{2} \rightarrow (2m+1)2^{m+1} = 2^7\sqrt{2} \rightarrow m = \frac{7}{2}, \; n = 2$$

$$2m + n = 9$$

۵

۸

$$\frac{\sqrt{x}+1}{\sqrt{x}-1} = 2 \rightarrow 2\sqrt{x} - 2 = \sqrt{x} + 1 \rightarrow \sqrt{x} = 3 \rightarrow x = 9$$

$$f'(x) = \frac{(\frac{1}{2\sqrt{x}})(\sqrt{x}-1) - (\frac{1}{2\sqrt{x}})(\sqrt{x}+1)}{(\sqrt{x}-1)^2} \longrightarrow f'(9) = -\frac{4}{3} \rightarrow$$ معکوس

$$\longrightarrow$$ شیب خط مماس $$= -\frac{3}{4}$$

۱

$$f(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1} \rightarrow \sqrt{x}y - y = \sqrt{x} + 1 \longrightarrow \sqrt{x}(y-1) = y + 1 \rightarrow \sqrt{x} = \frac{y+1}{y-1}$$

$$\rightarrow x = \left(\frac{y+1}{y-1}\right)^2 \longrightarrow y = \left(\frac{x+1}{x-1}\right)^2$$

$$f^{-1}(x) = \left(\frac{x+1}{x-1}\right)^2 \rightarrow (f^{-1})' = 2\left(\frac{x+1}{x-1}\right) \times \frac{-2}{(x-1)^2}$$

$$\longrightarrow (f^{-1}(2))' = 2 \times 3 \times (-2) = -12$$

۹

$$\frac{f(0) - f(-1)}{1} = -11 = 1 - (8)(-a+1) \longrightarrow \boxed{a = -\frac{1}{2}} \rightarrow x = -2a = 1$$

$$f'(x) = 3(x^2+1)^2(2x)(-\frac{1}{2}x + 1) + (x^2+1)^3(-\frac{1}{2}) \Rightarrow f'(1) = 12 - 4 = 8$$

۲

۱۰

$$y = (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = -\cos 2x \rightarrow \frac{f(\frac{\pi}{2}) - f(\frac{\pi}{4})}{\frac{\pi}{4}} = \frac{1 - 0}{\frac{\pi}{4}} = \frac{4}{\pi}$$

$$y = \sin x \cos 2x \rightarrow \frac{f(\frac{\pi}{2}) - f(\frac{\pi}{4})}{\frac{\pi}{4}} = \frac{1(-1) - 0}{\frac{\pi}{4}} = -\frac{4}{\pi}$$

$$\rightarrow \frac{-\frac{4}{\pi}}{\frac{4}{\pi}} = -1$$

۲