نام و نام خانوادگی: بهرنیا یلدا (نیان) .... ۲۵ .... پاسخنامه تشریحی تکلیف شماره ۲۳ .... کلاس دوازدهم تجربی A ....

$g(x) = f(\tan^2 x + \sqrt{2}\cos x) \longrightarrow g'(x) = (2(1+\tan^2 x)\tan x - \sqrt{2}\sin x)\, f'(\tan^2 x + \sqrt{2}\cos x)$ $x=\frac{\pi}{4} \Rightarrow g'(\frac{\pi}{4}) = (2(1+1)\times 1 - \frac{\sqrt{2}\times\sqrt{2}}{2})\, f'(1 + \sqrt{2}\times\frac{\sqrt{2}}{2})$ $\sqrt{3} = (4-1)\, f'(2) \longrightarrow f'(2) = \frac{\sqrt{3}}{3}$ ۲	۱
$g(x) = \frac{2}{2-\cos x} \longrightarrow g'(x) = \frac{-2\sin x}{(2-\cos x)^2} \longrightarrow g'(\frac{\pi}{6}) = \frac{+2\times\frac{1}{2}}{(2+\frac{\sqrt{3}}{2})^2} = \frac{4}{16+3+8\sqrt{3}} = \frac{4}{19+8\sqrt{3}}$ $f(x) = \frac{8+\cos^3 x}{4-\cos^2 x} \longrightarrow f'(x) = \frac{(3x-\sin x\times\cos^2 x)(4-\cos^2 x) - (2\times\sin x\times\cos x)(8+\cos^3 x)}{(4-\cos^2 x)^2}$ $f'(\frac{\pi}{6}) = \frac{(3\times -(-\frac{1}{2})\times(\frac{3}{4}))(4-\frac{3}{4}) - (\frac{\sqrt{3}}{2})\times(8-\frac{3\sqrt{3}}{8})}{(4-\frac{3}{4})^2} = \frac{(\frac{9}{8}\times\frac{13}{4}) - (4\sqrt{3} - \frac{9}{16})}{\frac{169}{16}} = \frac{\frac{117}{32} - 4\sqrt{3} + \frac{9}{16}}{\frac{169}{16}}$ $(4-\frac{3}{4})^2 \to \frac{13^2}{4^2} = \frac{169}{16}$ ۲	۲
$f'(x) \longrightarrow \lim_{x\to\frac{3}{4}^+} \frac{(4x-3)\sqrt{ax} - 0}{x-\frac{3}{4}} = 4\sqrt{ax} = 4\sqrt{\frac{3a}{4}} = 2\sqrt{3a}$ $\longrightarrow \lim_{x\to\frac{3}{4}^-} \frac{-(4x-3)\sqrt{ax} - 0}{x-\frac{3}{4}} = -4\sqrt{ax} = -4\sqrt{\frac{3a}{4}} = -2\sqrt{3a}$ $2\sqrt{3a} - (-2\sqrt{3a}) = 2\sqrt{6} \to 4\sqrt{3a} = 2\sqrt{6}$ $\sqrt{3a} = \frac{\sqrt{6}}{2} \to 3a = \frac{6}{4} \to a = \frac{1}{2}$ ۲	۳
$f'(4) = -a \ , \ f(4) = 2-4a$ $f(4)+f'(4) = -1 \to 2-4a-a = -1 \to 5a = 3 \to a = \frac{3}{5}$ $f(4) = -0/6$ $\frac{\frac{135}{32} - 4\sqrt{3}}{\frac{169}{16}} \to \frac{(135-128\sqrt{3})16}{2\times 32\times 169} = \frac{135-128\sqrt{3}}{338}$ $f'(\frac{\pi}{6}) - g'(\frac{\pi}{6}) = \frac{135-128\sqrt{3}}{338} - \frac{8}{19+8\sqrt{3}} = \frac{-169}{338} = -\frac{1}{2}$ ۲	۴
$y = \frac{x+5}{7}$, $y = \frac{ax-1}{3x+1} \to \frac{x+5}{7} = \frac{ax-1}{3x+1} \to 3x^2+16x+5 = 7ax-7$ وقتی دو نمودار بر هم مماس باشند یعنی معادلهٔ تلاقی آنها ریشهٔ مضاعف دارد ← $\Delta = 0$ $\to 3x^2 + (16-7a)x + 12 = 0$ چون حل تلاقی ناحیهٔ اول باید باشد ← X $\Delta = 0 \to (16-7a)^2 - 4\times 12\times 3 = 0 \to (16-7a)^2 = 144 \to 16-7a = 12 \to 7a = 4 \to a = \frac{4}{7}$; $16-7a = -12 \to 7a = 28 \to a = 4$ جواب قابل قبول ۲	۵

$f(2)=0 \to 8+2a-b=0 \to 8-24-b=0 \to b=-16$ $f'(2)=0 \to 12+a=0 \to a=-12$ $-16-(-12)=-4$	۶
$f(x)=\sin^n(x^2)\ ,\ \lim \frac{f(x)f'(x)}{(1-\cos x)^m}=32\sqrt{2}$ ($2^5\, 2^{1/2}=2^{5/5}$) $\to \lim_{x\to 0} \frac{2nx\cos(x^2)\sin^{2n-1}(x^2)}{(1-\cos x)^m}$ $f'(x)=n\sin^{n-1}(x^2)\times\cos(x^2)\times 2x$ $\sin x \sim x\ (x\to 0)$, $1-\cos x \sim \frac{x^2}{2}\ (x\to 0)$ $\to \frac{(2nx)(1)(x^2)^{2n-1}}{\frac{x^{2m}}{2^m}}=\frac{(2n\times 2^m)(x)(x^{4n-2})}{x^{2m}}$ x^{4n-1} $x^{2m}=x^{4n-1} \to 2m=4n-1$ ① $\to$ اعداد بدست آمده در ② در آن صدق می‌کند $\to 2m+n=7+2=9$ $n\cdot 2^{m+1}=2^{5/5}$ ② ; $n=2 \to 2^{m+2}=2^{5/5} \to m=3/5$	
$y=\frac{\sqrt{x}+1}{\sqrt{x}-1} \to \sqrt{x}y-y=\sqrt{x}+1 \to \sqrt{x}y-\sqrt{x}=y+1 \to \sqrt{x}(y-1)=y+1$ $\to \sqrt{x}=\frac{y+1}{y-1} \to x=\left(\frac{y+1}{y-1}\right)^2 \to f^{-1}(x)=\left(\frac{x+1}{x-1}\right)^2$ $f^{-1\prime}(x)=2\left(\frac{x+1}{x-1}\right)\left(\frac{(x-1)-(x+1)}{(x-1)^2}\right) \Rightarrow f^{-1\prime}(2)=2(3)(-2)=-12$	۸
$[-1,0]$ آهنگ تغییر متوسط $\to \frac{f(0)-f(-1)}{0-(-1)}=\frac{1-(8(1-a))}{1}=8a-7=-11 \to a=-\frac{1}{2}$ $f(x)=(x^2+1)^3(-\frac{x}{2}+1)$ $f'(x)=3(2x)(x^2+1)^2(-\frac{x}{2}+1)+-\frac{1}{2}(x^2+1)^3$ $f'(1)=3\times 2\times 4\times \frac{1}{2}-4=12-4=8$	۹
$y=\sin x\cos 2x \xrightarrow[\text{آهنگ}]{[\frac{\pi}{4},\frac{\pi}{2}]} \frac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{-1-0}{\frac{\pi}{4}}=-\frac{4}{\pi}$ ① $y=\sin^4 x-\cos^4 x \xrightarrow[\text{آهنگ}]{[\frac{\pi}{4},\frac{\pi}{2}]} \frac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{1-0}{\frac{\pi}{4}}=\frac{4}{\pi}$ ② $\frac{①}{②}=\frac{-\frac{4}{\pi}}{\frac{4}{\pi}}=-1$	۱۰