علیرضا اقطاعی:

۱۴٫۷۵

۱)

مشتق از ۲ طرف $\rightarrow g'(x) = (2\times\tan x\times(1+\tan^2 x) + (-\sqrt{2}\sin x))\times f'(\tan^2 x+\sqrt{2}\cos x)$

$\rightarrow g'(\frac{\pi}{4}) = ((2\times1\times2)+(-1))\times f'(1+1) \rightarrow \sqrt{3} = 3f'(2) \rightarrow \boxed{f'(2)=\frac{\sqrt{3}}{3}}$ ✓

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۲)

$f(x)=\frac{1+\cos^3 x}{4-\cos^2 x}$

$g(x)=\frac{2}{2-\cos x} \rightarrow \frac{4+2\cos x}{4-\cos^2 x}$

$f'(\frac{7\pi}{6})-2g'(\frac{7\pi}{6}) \rightarrow (f-2g)'(\frac{7\pi}{6}) \rightarrow \frac{1+\cos^3 x}{4-\cos^2 x}-\frac{2(4+2\cos x)}{4-\cos^2 x}$ [illegible]

[illegible]

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$\left(\frac{\cos^3 x-4\cos x}{4-\cos^2 x}\right)'(\frac{7\pi}{6}) \rightarrow \left(\frac{\cos x(\cos^2 x-4)}{4-\cos^2 x}\right)'(\frac{7\pi}{6}) \rightarrow (-\cos x)'(\frac{7\pi}{6})$

$\longrightarrow \sin\frac{7\pi}{6} \rightarrow \boxed{-\frac{1}{2}}$ ✓

۳) در این سوال نیم مماس ها قرینه هستند! چون فقط در ⊖ ضرب شدند پس اگر اختلاف $2\sqrt{6}$ است یعنی یکی $\sqrt{6}$ و دیگری $-\sqrt{6}$ است (ج)

نیم مماس راست $\rightarrow (4x-3)\sqrt{ax} \xrightarrow{'} 4\sqrt{ax}+\frac{a(4x-3)}{2\sqrt{ax}}$

$\xrightarrow{x=\frac{3}{4}} 4\sqrt{\frac{3a}{4}}=\sqrt{6} \rightarrow 2\sqrt{3a}=\sqrt{6} \rightarrow \sqrt{12a}=\sqrt{6} \rightarrow 12a=6 \rightarrow \boxed{a=\frac{1}{2}}$ ✓

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۴)

مماس بود $\rightarrow$ $f(a)=g(a)$, $f'(a)=g'(a)$ $\Rightarrow f(4)=g(4) \rightarrow f(4)=2-4a$ $\longleftarrow \boxed{g(x)=-ax+2}$

$f'(4)=g'(4) \rightarrow f'(4)=-a$ $\longleftarrow \boxed{g'(x)=-a}$

$\rightarrow f(4)+f'(4) \rightarrow 2-5a=-1 \rightarrow 5a=3 \rightarrow \boxed{a=0.6}$ $\quad \boxed{f'(4)=-0.6}$ ✓

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۵) اگر α نقطه مماس باشد،

$\boxed{7y-x=5}$ $\xrightarrow{\text{شیب}} \frac{-a}{b} \rightarrow \frac{-(-1)}{7}=\boxed{\frac{1}{7}}$ $\quad \boxed{y=\frac{x+5}{7}}$

$y=\frac{ax-1}{3x+1} \xrightarrow{'} \frac{a+3}{(3x+1)^2}$

$\rightarrow \frac{\alpha+5}{7}=\frac{a\alpha-1}{3\alpha+1}$ و $\frac{a+3}{(3\alpha+1)^2}=\frac{1}{7}$

$\rightarrow 3\alpha^2+16\alpha+5=7a\alpha-7$

$\rightarrow \boxed{7a\alpha=3\alpha^2+16\alpha+12}$ I

$\rightarrow 7a+21=9\alpha^2+6\alpha+1$ III

$\rightarrow 7a=9\alpha^2+6\alpha-20 \rightarrow \boxed{7a\alpha=9\alpha^3+6\alpha^2-20\alpha}$ II

$I, II \rightarrow 9\alpha^3+6\alpha^2-20\alpha=3\alpha^2+16\alpha+12$

$\rightarrow 9\alpha^3+3\alpha^2-36\alpha-12=0$

$\rightarrow 3\alpha^3+\alpha^2-12\alpha-4=0$

درس $\boxed{\alpha=2}$

III $7a+21=36+12+1 \rightarrow \boxed{a=4}$ ✓

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(۶) $f'(a) = 3a^2 + a$

$f(2) = 0 \rightarrow 8 + 2a - b = 0$

$f'(2) = 0 \rightarrow 12 + a = 0 \xrightarrow{\times 2} 24 + 2a = 0$ $\Big\} \rightarrow -16 - b = 0 \rightarrow \boxed{b = -16}$ (۲)

$\rightarrow \boxed{a = -12} \rightarrow b - a \rightarrow \boxed{-4}$ ✓

(۷) $f(x) = (x)^{n+2} \rightarrow$ مشتق‌گیری $\rightarrow f'(x) = (n+2)(x)^{n+1}$ (۰/۵)

$\lim_{x \to 0} \frac{f(x) f'(x)}{(1 - \cos x)^m} \xrightarrow{\text{مشتق‌گیری}} \frac{x^{n+2} \times (n+2) \times x^{n+1}}{1 - m\cos x}$

$(1 - \cos x)^m \rightarrow 1 - m\cos x$

$\rightarrow \frac{x^{2n+3} \times (n+2)}{1 - m} \rightarrow \frac{n+2}{m} = -\frac{3}{2} \rightarrow n = -\frac{3}{2}$

(جواب پایین صفحه)

(۱) $f \rightarrow (9, 2)$

$\frac{\sqrt{x} + 1}{\sqrt{x} - 1} = 2 \rightarrow \sqrt{x} + 3 = 2\sqrt{x} - 2 \rightarrow \boxed{x = 9}$

$f \rightarrow \frac{\sqrt{x} - 1}{2\sqrt{x}} - \frac{\sqrt{x} + 1}{2\sqrt{x}} \rightarrow \frac{-2}{2\sqrt{x}} \rightarrow \boxed{\frac{-1}{\sqrt{x}}}$ (۰/۵)

(جواب پایین صفحه)

$x = 9 \rightarrow -\frac{1}{3}$

$(f^{-1}(1))' \rightarrow -3$ جواب ✗

(۹) $-1 \xrightarrow{f} 1 - a \rightarrow 1 - 8a$

$0 \xrightarrow{f} 1 \; (1) = 1$ $\Big\} \; \frac{(1 - 8a) - 1}{1} = 11 \rightarrow \boxed{a = -\frac{1}{2}}$

(۱/۲۵)

$x = -2a \rightarrow \boxed{x = 1} \rightarrow f' \rightarrow 3 \times 2x \times (x^2 + 1)^2 (ax + 1) + a(x^2 + 1)^3$

$\rightarrow \frac{3 \times 2 \times 4}{24} + 8a \rightarrow 24 - 4 = 20$ ~~$= 20$~~ $\quad 12 - 4 = 8$

(۱۰) $f \rightarrow \sin a \cos 2x$

$g \rightarrow \sin^4 x - \cos^4 x$

$\hookrightarrow -\cos 2x$

$\frac{\pi}{4} \xrightarrow{f} 0$

$\frac{\pi}{2} \xrightarrow{f} -1$ $\rightarrow \frac{0 - (-1)}{\frac{-\pi}{4}} \rightarrow \frac{-4}{\pi}$

$\frac{\pi}{4} \rightarrow 0$

$\frac{\pi}{2} \rightarrow 1$ $\rightarrow \frac{0 - 1}{\frac{-\pi}{4}} = \frac{4}{\pi}$

$\frac{\text{اولی}}{\text{دومی}} = -1$ ✓ (۲)

سوال ۱۷

$$\lim_{x\to 0}\frac{\sin^n(x^2)\times n\sin^{n-1}(x^2)\times \cos(x^2)\times(2x)}{\left(2\sin^2\frac{x}{2}\right)^m}\xrightarrow{\text{هم‌ارزی}}\lim_{x\to 0}\frac{(x^2)^n\, n\,(x^2)^{n-1}\times 2x}{\left(2\left(\frac{x}{2}\right)^2\right)^m}=$$

$$\lim_{x\to 0}\frac{2n\,x^{2n+2n-2+1}}{\left(\frac{x^2}{2}\right)^m}=\lim_{x\to 0}\frac{2n\,x^{4n-1}}{\frac{1}{2^m}x^{2m}}=\lim_{x\to 0}2^{m+1}n\,x^{4n-2m-1}$$

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$$\begin{cases}4n-2m-1=0\longrightarrow n=\frac{2m+1}{4}\\ 2^{m+1}n=32\sqrt{2}\end{cases}\longrightarrow 2^{m+1}\times\frac{2m+1}{4}=32\sqrt{2}\longrightarrow(2m+1)2^{m+1}=2^7\sqrt{2}\longrightarrow m=\frac{7}{2},\quad n=2$$

$$2m+n=9$$

سوال ۱۸

$$f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1}\rightarrow\sqrt{x}y-y=\sqrt{x}+1\longrightarrow\sqrt{x}(y-1)=y+1\rightarrow\sqrt{x}=\frac{y+1}{y-1}\rightarrow x=\left(\frac{y+1}{y-1}\right)^2\longrightarrow y=\left(\frac{x+1}{x-1}\right)^2$$

$$f^{-1}(x)=\left(\frac{x+1}{x-1}\right)^2\rightarrow(f^{-1})'=2\left(\frac{x+1}{x-1}\right)\times\frac{-2}{(x-1)^2}\longrightarrow\left(f^{-1}(2)\right)'=2\times3\times(-2)=-12$$