نام و نام خانوادگی ..دانیال ... ~~تجربت~~ ...... ۱۵ پاسخنامه تشریحی تکلیف شماره ..۲۳.. کلاس ..دوازدهم ریاضی..۸......

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$g'(x)=(2\tan x\times(1+\tan^2 x)-\sqrt{2}\sin x)\times f'(\tan^2 x+\sqrt{2}\cos x)$

$g'(\frac{\pi}{4}) = 3f'(2)=\sqrt{3} \longrightarrow f'(2)=\boxed{\frac{\sqrt{3}}{3}}$ ✓

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$\to (f-2g)'=\left(\frac{8+\cos^3 x}{4-\cos^2 x}-\frac{4}{2-\cos x}\right)'=\left(\frac{(\cos x+2)(\cos^2 x-2\cos x+4)}{(2-\cos x)(2+\cos x)}-\frac{4}{2-\cos x}\right)'=$

$\left(\frac{\cos x(\cos x-2)}{-(\cos x-2)}\right)'=-(\cos x)'=\sin x \xrightarrow{x=7\pi/6} \sin(\frac{7\pi}{6})=\frac{-1}{2}$

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$f(x)=\frac{8+\cos^3 x}{4-\cos^2 x} \longrightarrow f'(x)=\frac{(3\cos^2 x)\times(-\sin x)(4-\cos^2 x)-(8+\cos^3 x)(-2\cos x)(-\sin x)}{(4-\cos^2 x)^2}$

$g(x)=\frac{2}{2-\cos x} \to g'(x)=\frac{-2(\sin x)}{(2-\cos x)^2}$

$f'(\frac{7\pi}{6})-2g'(\frac{7\pi}{6})=(f-2g)'(\frac{7\pi}{6})$

$f'(\frac{7\pi}{6})=\frac{153-128\sqrt{3}}{338}$ $\quad g'(\frac{7\pi}{6})=\frac{76-32\sqrt{3}}{169}=\frac{152-64\sqrt{3}}{338}$

$f'(\frac{7\pi}{6})-2g'(\frac{7\pi}{6})=\frac{153-128\sqrt{3}}{338}+\frac{-304+128\sqrt{3}}{338}=\boxed{-\frac{151}{338}}$

۰/۸

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$f'_+(\frac{3}{4})=4\sqrt{ax}+(4x-3)\times\frac{a}{2\sqrt{ax}}$

$f'_-(\frac{3}{4})=-4\sqrt{ax}-(4x-3)\times\frac{a}{2\sqrt{ax}}$

$f'_+(\frac{3}{4})-f'_-(\frac{3}{4})=8\sqrt{ax}+(8x-6)\times\frac{a}{2\sqrt{ax}}=4\sqrt{3a}=2\sqrt{6}$

$\sqrt{3a}=\frac{\sqrt{6}}{2} \to 3a=\frac{6}{4} \to \boxed{a=\frac{1}{2}}$ ✓

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$y=-ax+2$

$f(4)=-4a+2$

$f'(4)=-a$

$f(4)+f'(4)=-5a+2=-1 \to a=\frac{3}{5}$

$\boxed{f'(4)=-\frac{3}{5}}$ ✓

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$y=\frac{x+5}{7}$ $\qquad \xrightarrow{\Delta=0} (16-7a)^2-4(3)(12)=0 \to \begin{cases} a=\frac{4}{7} \\ a=4 \end{cases}$

۱/۵

$\frac{ax-1}{3x+1}=\frac{x+5}{7} \to 7ax-7=3x^2+16x+5 \to 3x^2+(16-7a)x+12=0$

$\Delta=(16-7a)^2-4(3)(12)=0 \longrightarrow 49a^2-224a+112=0 \longrightarrow \begin{cases} a=\frac{8}{7} \\ a=\frac{24}{7} \end{cases}$

$3x^2+(16-7a)x+12 \xrightarrow{\text{مضاعف}} x=\frac{-b}{2a}=\frac{-16+7a}{6} \to \begin{cases} a=4 \to x=2 \quad ✓ \\ a=\frac{4}{7} \to x=-2 \quad ✗ \end{cases}$

$f(2) = 8 + 2a - b = 0 \rightarrow 2a - b = -8 \rightarrow b = -16$ $f'(2) = 3x^2 + a = 12 + a = 0 \rightarrow a = -12$ $b - a = -16 + 12 = \boxed{-4}$ 4 $\lim\limits_{x\to 0} \frac{2n x^{2n+2n-2+1}}{(\frac{x^2}{2})^m} = \lim\limits_{x\to 0} \frac{2n x^{4n-1}}{\frac{1}{2^m} x^{2m}} = \lim\limits_{x\to 0} 2^{m+1} n x^{4n-2m-1}$ ⟵ حاصل حد عدد غیرصفر متناهی غیرصفر ← توان صفر شوند $\begin{cases} 4n - 2m - 1 = 0 \rightarrow n = \frac{2m+1}{4} \\ 2^{m+1} n = 32\sqrt{2} \end{cases} \rightarrow 2^{m+1} \times \frac{2m+1}{4} = 32\sqrt{2} \rightarrow (2m+1) 2^{m+1} = 2^7\sqrt{2} \rightarrow m = \frac{7}{2},\ n = 2$ $\quad 2m + n = 9$	۶
$f(x) = \sin^n(x^2) \rightarrow f'(x) = n \sin^{n-1}(x^2) \times \cos x \times 2x$ $\quad$ 1 $\lim\limits_{x\to 0} \frac{f(x) f'(x)}{(1-\cos x)^m} = \frac{\sin^n(x^2) \times n \times \sin^{n-1}(x^2) \times \cos x \times 2x}{(1-\cos x)^m}$ $\rightarrow \frac{x^{2n} \times n \times x^{2n-2} \times (1 - \frac{x^2}{2}) \times 2x}{(1-(1-\frac{x^2}{2}))^m} = \frac{2 \times n \times x^{4n-1} \times (1-\frac{x^2}{2})}{(\frac{x^2}{2})^m}$ $2^{m+1} \times n \times x^{4n-2m-1} \times (1 \cancel{- \frac{x^2}{2}}) = 2$ $n \times x^{4n-2m-1} \times (1 - \frac{x^2}{2}) = 2^{\frac{9}{2} - m} \rightarrow \begin{cases} \frac{9}{2} - m = 0 \rightarrow m = \frac{9}{2} \\ 4n - 2m - 1 = 0 \rightarrow n = \frac{5}{2} \end{cases}$ $\quad \boxed{2m + n = \frac{23}{2}}$	۷
0 $f(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1} \rightarrow f'(x) = \frac{\frac{1}{2\sqrt{x}}(\sqrt{x}-1) - (\sqrt{x}+1)\frac{1}{2\sqrt{x}}}{(\sqrt{x}-1)^2} = \frac{-\frac{1}{\sqrt{x}}}{x - 2\sqrt{x} + 1}$ $f''(x) = \frac{-\frac{1}{2}(x)^{-\frac{3}{2}}(x - 2\sqrt{x} + 1) - (-\frac{1}{\sqrt{x}})(1 - \frac{1}{\sqrt{x}})}{(x - 2\sqrt{x} + 1)^2} = \frac{\frac{1}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}}}{(x - 2\sqrt{x} + 1)^2}$ $f''(2) = \frac{\frac{1}{2\sqrt{2}} - \frac{1}{4\sqrt{2}}}{(3 - 2\sqrt{2})^2} = \frac{\frac{1}{4\sqrt{2}}}{17 - 12\sqrt{2}} = \boxed{\frac{1}{68\sqrt{2} - 96}}$ $f(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1} \rightarrow \sqrt{x}y - y = \sqrt{x} + 1 \rightarrow \sqrt{x}(y-1) = y + 1 \rightarrow \sqrt{x} = \frac{y+1}{y-1} \rightarrow x = (\frac{y+1}{y-1})^2 \rightarrow y = (\frac{x+1}{x-1})^2$ $f^{-1}(x) = (\frac{x+1}{x-1})^2 \rightarrow (f^{-1})' = 2(\frac{x+1}{x-1}) \times \frac{-2}{(x-1)^2} \rightarrow (f^{-1}(2))' = 2 \times 3 \times (-2) = -12$	۸
$f(-1) = 8(1-a) = 8 - 8a$ $\quad \frac{1 - 8 + 8a}{0 + 1} = 8a - 7 = -11 \rightarrow a = -\frac{1}{2}$ $f(0) = 1$ $f(x) = (x^2+1)^3(-\frac{1}{2}x + 1)$ $f'(x) = 3(x^2+1)^2 \times 2x \times (-\frac{1}{2}x + 1) + (x^2+1)^3 \times (-\frac{1}{2})$ $f'(-2a) = f'(1) = \boxed{8}$ 2	۹
$y = \sin x \cos 2x \rightarrow \begin{matrix} \sin\frac{\pi}{4}\cos\frac{\pi}{2} = 0 \\ \sin\frac{\pi}{2}\cos\pi = -1 \end{matrix}$ $\quad \frac{-1 - 0}{\frac{\pi}{2} - \frac{\pi}{4}} = -\frac{4}{\pi}$ $y = \sin^4 x - \cos^4 x = -\cos 2x \rightarrow \begin{matrix} -\cos\frac{\pi}{2} = 0 \\ -\cos\pi = 1 \end{matrix}$ $\quad \frac{1 - 0}{\frac{\pi}{2} - \frac{\pi}{4}} = \frac{4}{\pi}$ $\frac{-\frac{4}{\pi}}{\frac{4}{\pi}} = \boxed{-1}$ 2	۱۰