$g'(2) = f'(\tan^2 2 + \sqrt{2}\cos 2) \times \left(2\tan 2\left(\frac{1}{\cos^2 2}\right) - \sqrt{2}\sin 2\right)$

$g'\left(\frac{\pi}{4}\right) = f'(1+\frac{\sqrt{2}}{2}) \times (2 \times 1)\left(1+1-\frac{\sqrt{2}}{\sqrt{2}}\right)$

$\sqrt{2} = f'(2) \times (2) \Rightarrow f'(2) = \frac{\sqrt{2}}{2}$ (2)

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(3) (جواب باسين للغه)

$f(2) = (2-\cos 2) + \cos 2\, g(2)$

$\Rightarrow f'(2) = 2\sin 2 + \cos 2\, g'(2) - \sin 2\, g'(2)$

$\rightarrow f'\left(\frac{\pi}{4}\right) = \frac{1}{2}\left(\frac{2-\sqrt{2}}{2}\right) g'\left(\frac{\pi}{4}\right)$

$f'\left(\frac{\pi}{4}\right) - 2g'\left(\frac{\pi}{4}\right) = \frac{1}{2} - \left(\frac{2-\sqrt{2}}{2}\right) g'\left(\frac{\pi}{4}\right)$

$= \frac{1}{2} - \left(\frac{2-\sqrt{2}}{2}\right)\left(\frac{-1}{2\left(\frac{2}{\sqrt{2}}-1\right)}\right)$

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(4) $f(2) = |4-2|\, a\sqrt{2}$

$\Rightarrow \sqrt{\frac{4}{2}}\, a = 2\sqrt{2}$

$f'\left(\frac{\pi}{2}\right) = (4-2)\sqrt{a}\,\frac{1}{2\sqrt{2}} \Rightarrow \sqrt{\frac{2}{2}}\, a = \sqrt{2}$

$f\left(\frac{\pi}{2}\right) = -(4-2)\sqrt{a}\,2\sqrt{2}$

$\sqrt{\frac{\pi^2}{4}+1} = \sqrt{2} \quad \pi a + 2 = 2 \Rightarrow a = \frac{1}{2}$ (2)

$g = -2a + 2 \qquad f'(2) = -a \qquad f(2) = -2a + 2$

$f(2) + f'(2) = -3a + 2 = -1 \qquad 3a = 3$

(4)

$a = \frac{3}{5} \qquad f'(2) = -\frac{3}{5}$ (2)

$g = \frac{2}{V} + \frac{5}{V}$ $\quad f'(2) = \frac{1}{V}$ $\quad f(2) = \frac{2}{V} + \frac{5}{V}$

$f(2) = \frac{ax-1}{4x+1}$ $\quad \frac{a+4}{(4x+1)^2} = \frac{1}{V}$ $\quad \frac{V(ax-1)}{4x+1} = x-5$

$f'(x) = \frac{ax+4}{(4x+1)^2}$ $\quad a = \frac{(4x+1)^2}{V} - 4$ (۵)

(۲)

$Vax - V = 4x^2 + 8x + 5 \quad x = 2 \quad a = 6$ ✓

$f(2) = 0 \quad f'(2) = 0 \quad b = -16 \quad a = -12$

$f(x) = x^2 + ax \rightarrow a = -12$ (۶)

(۲)

$f(2) = 2^2 - a(2) - b \rightarrow 8 - 24 - b = 0 \rightarrow b = -16$

$b - a = -16 + 12 = -4$ ✓

$\lim_{x \to 0} \frac{f(x)\, f'(x)}{(1-\cos x)^m} = 4\sqrt{2}$ (۷)

$\rightarrow \frac{x^{2n} \times 2^{2n-2} \times \left(1 - \frac{x^2}{2}\right)^4}{\left(1 - \left(1 - \frac{x^2}{2}\right)\right)^m}$

$m = 2n - \frac{1}{2}$

$2n^m = 2 \Rightarrow m = \frac{7}{2} \quad n = 2$

$2m + n = 2\left(\frac{7}{2}\right) + 2 = 9$ ✓

(۲)

$f'\left(\frac{7\pi}{6}\right) - 2g'\left(\frac{7\pi}{6}\right) = (f - 2g)'\left(\frac{7\pi}{6}\right)$ سوال ۴

$\rightarrow (f-2g)' = \left(\frac{1 + \cos^2 x}{4 - \cos^2 x} - \frac{4}{2 - \cos x}\right) = \left(\frac{(\cos x + 2)(\cos^2 x - 2\cos x + 4)}{(2 - \cos x)(2 + \cos x)} - \frac{4}{2 - \cos x}\right) =$

$\left(\frac{\cos(\cos x - 2)}{-(\cos x - 2)}\right)' = -(\cos x)' = \sin x \xrightarrow{x = \frac{7\pi}{6}} \sin\left(\frac{7\pi}{6}\right) = \frac{-1}{2}$

$f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1} \to \sqrt{x}y-y=\sqrt{x}+1 \to \sqrt{x}(y-1)=y+1 \to \sqrt{x}=\frac{y+1}{y-1} \to x=\left(\frac{y+1}{y-1}\right)^2 \to y=\left(\frac{x+1}{x-1}\right)^2$

$f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1} \to f'(x)=\frac{-2}{(\sqrt{x}-1)^2}$ $\quad f(a)=2$ $\quad f^{-1}(2)=9$

$f(x)=\frac{x+1}{x-1} \Rightarrow f'(x)=\frac{-2}{(x-1)^2}$ (1)

$f'(9)=\frac{-2}{8}=\frac{-1}{4}$ $\quad f^{-1}(x)=\left(\frac{x+1}{x-1}\right)^2 \to (f^{-1})'=2\left(\frac{x+1}{x-1}\right)\times\frac{-2}{(x-1)^2}$

$f^{-1}(2)=-4$ $\quad \to (f^{-1}(2))'=2\times 4\times(-4)=-32$

$\frac{f(a)-f(-1)}{a-(-1)}=\frac{1-(2)^{10}}{(1-a)}$ $\Rightarrow$ (2)

$\left.\begin{array}{l} 4a=-4 \\ a=\frac{-1}{2} \end{array}\right\}$ $f'(1)=10\times 2x(x^2+1)^9\left(1-\frac{1}{2}x\right)$ (3)

$-\frac{1}{2}(x^2+1)^{10}=12-8=4$ (4) ✓

$y=\sin x\cos 2x$ (10)

$y=(\sin^2\frac{x}{2}+\cos^2\frac{x}{2})(\sin^2 x-\cos^2 x)-\cos 2x$

$\frac{(\sin\frac{\pi}{2}\cdot\cos 2x)-(\sin(\frac{\pi}{2})\cos\frac{\pi}{2})}{\frac{\pi}{2}}=\frac{1}{-1}=-1$ (5) ✓