۷/۷۵

نام و نام خانوادگی ...... فرید غفاری ...... پاسخنامه تشریحی تکلیف شماره ...... کلاس ......

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$$g'(x) = \left(2\tan x\,(1+\tan^2 x) - \sqrt{2}\sin x\right) f'\left(\tan^2 x + \sqrt{2}\cos x\right)$$

$$\xrightarrow{x=\pi/4} g'\left(\tfrac{\pi}{4}\right) = 3f'(2) = \sqrt{3} \longrightarrow f'(2) = \frac{1}{\sqrt{3}} \longrightarrow f'(2) = \frac{\sqrt{3}}{3}$$

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$$f(x) = \frac{8+\cos^3 x}{4-\cos^2 x} \Rightarrow f'(x) = \frac{\left((-3\sin^2 x)\times(4-\cos^2 x)\right) - \left((2\sin x)\times(8+\cos^3 x)\right)}{(4-\cos^2 x)^2}$$

۰ (جواب پایین صفحه)

$$g(x) = \frac{2}{2-\cos x} \Rightarrow g'(x) = \frac{-2\sin x}{(2-\cos x)^2} = \frac{-2\sin\frac{\cancel{V}\pi}{4}}{\left(2-\cos\frac{V\pi}{4}\right)^2} = \frac{1}{\left(3+\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{2+2\sqrt{3}+\frac{3}{4}}$$

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$$f(x) = |4x-3|\sqrt{ax} \quad \begin{cases} \xrightarrow{\frac{3}{4}^+} (4x-3)(\sqrt{ax}) \Rightarrow 4(\sqrt{ax}) + (\cancel{4x-3})\frac{a}{2\sqrt{ax}} = 4\sqrt{\frac{3a}{4}} \\ \xrightarrow{\frac{3}{4}^-} (-4x+3)(\sqrt{ax}) \Rightarrow -4(\sqrt{ax}) + (-4x+3)\frac{a}{2\sqrt{ax}} = -4\sqrt{\frac{3a}{4}} \end{cases}$$

$$4\sqrt{\frac{3a}{4}} - \left(-4\sqrt{\frac{3a}{4}}\right) = 2\sqrt{6} \Rightarrow \cancel{2}4\sqrt{\frac{3a}{4}} = \cancel{2}\sqrt{6} \Rightarrow 4\sqrt{\frac{3a}{4}} = 6$$

$$16 \times \frac{3a}{4} = 36 \Rightarrow 12a = 36 \Rightarrow \boxed{a=3}$$

$$\sqrt{\frac{3}{4}a} = \frac{\sqrt{6}}{4} \rightarrow \frac{3}{4}a = \frac{6}{16} \rightarrow a = \frac{1}{2}$$

۱/۲۵

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$$y = -ax+2 \xrightarrow{x=4} -4a+2$$

$$y' = -a$$

$$f(4) + f'(4) = -1 \Rightarrow -4a+2-a = -1 \Rightarrow -5a = -3$$

$$f'(4) = -a = -\frac{3}{5} = -0.6 \qquad \boxed{a = \frac{3}{5}}$$

۱/۷۵

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$$7y - x = 5 \Rightarrow 7y = x+5 \Rightarrow y = \frac{x}{7} + \frac{5}{7}$$

۰/۵

$$\rightarrow 7ax - 7 = 3x^2 + x + 15x + 5 \rightarrow 3x^2 + (16-7a)x + 12 = 0$$

$$f(x) = \frac{ax-1}{3x+1} = \frac{x+5}{7} \qquad \xrightarrow{\Delta=0} (16-7a)^2 - 4(3)(12) = 0 \rightarrow \begin{cases} a = \frac{4}{7} \\ a = 4 \end{cases}$$

$$f'(x) = \frac{-ax+a+1}{3x+1} = \frac{1}{7}$$

$$3x^2 + (16-7a)x + 12 \xrightarrow{\text{مماس}} x = \frac{-b}{2a} = \frac{-16+7a}{6} \rightarrow \begin{cases} a = 4 \rightarrow x = 2 & \checkmark \\ a = \frac{4}{7} \rightarrow x = -2 & \times \end{cases}$$

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فرید غلامی

$f(x)=x^3+ax-b$ $8+2a-b=0 \Rightarrow 2a-b=-8 \Rightarrow 2(\overset{-24}{-12})-b=-8 \Rightarrow -b=16$ $b=-16$ $f'(x)=3x^2+a \Rightarrow 3\times 2^2+a=0 \Rightarrow a=-12$ ۰/۷۵ $a+b=-12-16=\boxed{-28}$ $b-a=-16-(-12)=-4$	۶
$\lim\limits_{x\to 0}\frac{\sin^n(x^2)\times n\sin^{n-1}(x^2)\times \cos(x^2)\times(2x)}{(2\sin^2\frac{x}{2})^m} \xrightarrow{\sin\approx} \lim\limits_{x\to 0}\frac{(x^2)^n n(x^2)^{n-1}\times 2x}{(2(\frac{x}{2})^2)^m}=$ $\lim\limits_{x\to 0}\frac{2nx^{2n+2n-2+1}}{(\frac{x^2}{2})^m}=\lim\limits_{x\to 0}\frac{2nx^{4n-1}}{\frac{1}{2^m}x^{2m}}=\lim\limits_{x\to 0}2^{m+1}n\,x^{4n-2m-1}$ ⟶ حاصل حد عدد غیرمتناهی غیرصفر ← توان صفر $\begin{cases}4n-2m-1=0 \longrightarrow n=\frac{2m+1}{4}\\ 2^{m+1}n=32\sqrt{2}\end{cases} \longrightarrow 2^{m+1}\times\frac{2m+1}{4}=32\sqrt{2} \rightarrow (2m+1)2^{m+1}=2^7\sqrt{2} \longrightarrow m=\frac{7}{2}$ $2m+n=9$ $n=2$ ۰	۷
$f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow f'(x)=\frac{\frac{1}{2\sqrt{x}}\times(\sqrt{x}-1)-\frac{1}{2\sqrt{x}}\times(\sqrt{x}+1)}{(\sqrt{x}-1)^2}=\frac{\frac{-2}{2\sqrt{x}}}{(\sqrt{x}-1)^2}$ ۰ $f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1} \rightarrow \sqrt{x}y-y=\sqrt{x}+1 \longrightarrow \sqrt{x}(y-1)=y+1 \rightarrow \sqrt{x}=\frac{y+1}{y-1} \rightarrow x=(\frac{y+1}{y-1})^2 \rightarrow y=(\frac{x+1}{x-1})^2$ $f^{-1}(x)=(\frac{x+1}{x-1})^2 \rightarrow (f^{-1})'=2(\frac{x+1}{x-1})\times\frac{-2}{(x-1)^2} \longrightarrow (f^{-1}(2))'=2\times3\times(-2)=-12$	۸
$f(x)=(x^2+1)^3(ax+1) \Rightarrow f(0)=(1)(1)=1$ (جواب پاسخنامه) $f(-1)=(2)\times(-a+1)=-2a+2$ $\frac{(-2a+2)-1}{0-(-1)}=-11 \Rightarrow -2a+1=-11 \Rightarrow -2a=-12 \Rightarrow a=6$ ۰ $x=-2a=-12$ $f'(x)=3(2x)(x^2+1)^2\times(6x+1)+6(x^2+1)=\overbrace{3\times-12\times145\times-71}^{370620}+\overbrace{6(145)}^{870}=371490$	۹
$y=\sin x\cos 2x \xrightarrow{\frac{\pi}{4}} \sin\frac{\pi}{4}\cos\frac{2\pi}{4} \Rightarrow \frac{\sqrt{2}}{2}\times 0=0$ $\xrightarrow{\frac{\pi}{2}} \sin\frac{\pi}{2}\cos\frac{2\pi}{2} \Rightarrow 1\times-1=-1$ $y=\sin^4x-\cos^4x \Rightarrow (\overset{1}{\sin^2x+\cos^2x})(\sin^2x-\cos^2x) \xrightarrow{\frac{\pi}{4}} (\frac{\sqrt{2}}{2})^2-(\frac{\sqrt{2}}{2})^2=0$ ۲ $\xrightarrow{\frac{\pi}{2}} (1)^2-(0)=1$ $\frac{-1-0}{\frac{\pi}{4}-\frac{\pi}{2}}=\frac{-1}{\frac{1}{1}}=\boxed{-1}$ ✓	۱۰

$\frac{1-0}{\frac{\pi}{4}-\frac{\pi}{2}}$ $\frac{1}{1}$

سوال ۱۲

$$f'\left(\frac{7\pi}{6}\right)-2g'\left(\frac{7\pi}{6}\right)=(f-2g)'\left(\frac{7\pi}{6}\right)$$

$$\rightarrow (f-2g)'=\left(\frac{8+\cos^3 x}{4-\cos^2 x}-\frac{4}{2-\cos x}\right)'=\left(\frac{(\cos x+2)(\cos^2 x-2\cos x+4)}{(2-\cos x)(2+\cos x)}-\frac{4}{2-\cos x}\right)'=$$

$$\left(\frac{\cos x(\cos x-2)}{-(\cos x-2)}\right)'=-(\cos x)'=\sin x\xrightarrow{x=7\pi/6}\sin\left(\frac{7\pi}{6}\right)=\frac{-1}{2}$$

سوال ۱۹

$$\underset{[-1,0]}{\text{آهنگ متوسط تغییر}}=\frac{f(0)-f(-1)}{0-(-1)}=\frac{1-(8(-a+1))}{1}=\frac{1-(-8a+8)}{1}=8a-7=-11\longrightarrow\begin{cases}a=\frac{-1}{2}\\ x=1\end{cases}$$

$$f'(x)=3(2x)(x^2+1)^2(ax+1)+a(x^2+1)^3\xrightarrow[a=\frac{-1}{2}]{x=1}3(2\times1)(1+1)^2\left(\frac{-1}{2}\times1+1\right)+\frac{-1}{2}(1+1)^3=8$$