A سری ۳ — مشتق بهرامی — ۲۰

1

$$g'(x)=\left(2\tan x(\tan^2x+1)-\sqrt{2}\sin x\right)f'\left(\tan^2x+\sqrt{2}\cos x\right)$$

$$x=\frac{\pi}{4}: g'\left(\frac{\pi}{4}\right)=\left(2\underbrace{\tan\frac{\pi}{4}}_{1}\left(\underbrace{\tan^2\frac{\pi}{4}}_{1}+1\right)-\sqrt{2}\underbrace{\sin\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}}\right)f'\left(\underbrace{\tan^2\frac{\pi}{4}}_{1}+\sqrt{2}\underbrace{\cos\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}}\right)=(4-1)f'(2)=f'(2)$$

$$\Rightarrow f'(2)=\frac{\sqrt{3}}{3}$$

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2

$$f'(x)-2g'(x)=(f-2g)'(x),\quad f(x)=\frac{8+\cos^3x}{4-\cos^2x}=\frac{(\cos x+2)(\cos^2x-2\cos x+4)}{-(\cos x+2)(\cos x-2)}=\frac{\cos^2x-2\cos x+4}{2-\cos x}$$

$$(f-2g)(x)=\frac{\cos^2x-2\cos x+4}{2-\cos x}-2\left(\frac{2}{2-\cos x}\right)=\frac{\cos^2x-2\cos x+4-4}{2-\cos x}=\frac{\cos^2x-2\cos x}{2-\cos x}=\frac{-\cos x(2-\cos x)}{2-\cos x}$$

$$\Rightarrow (f-2g)(x)=-\cos x\Rightarrow (f-2g)'(x)=\sin x\Rightarrow \boxed{(f-2g)'\left(\frac{7\pi}{6}\right)=\sin\frac{7\pi}{6}=-\frac{1}{2}}$$

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3

$$f(x)=\begin{cases}(4x-3)\sqrt{ax}; & x\geq\frac{3}{4}\\ (3-4x)\sqrt{ax}; & x\leq\frac{3}{4}\end{cases}\Rightarrow f'(x)=\begin{cases}4\sqrt{ax}+\left(\frac{a}{2\sqrt{ax}}\right)(4x-3); & x>\frac{3}{4}\\ -4\sqrt{ax}+\left(\frac{a}{2\sqrt{ax}}\right)(3-4x); & \frac{3}{4}>x>0\end{cases}$$

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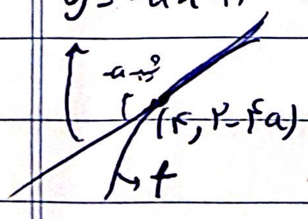
$$f'_+\left(\frac{3}{4}\right)=4\sqrt{a\times\frac{3}{4}}+0,\quad f'_-\left(\frac{3}{4}\right)=-4\sqrt{a\times\frac{3}{4}}+0\Rightarrow \left|f'_+\left(\frac{3}{4}\right)-f'_-\left(\frac{3}{4}\right)\right|=\left|4\sqrt{\frac{3a}{4}}-\left(-4\sqrt{\frac{3a}{4}}\right)\right|$$

$$\Rightarrow\left|f'_+\left(\frac{3}{4}\right)-f'_-\left(\frac{3}{4}\right)\right|=|\underbrace{2\sqrt{3a}+2\sqrt{3a}}_{\text{جمع رادیکال‌ها همواره نامنفی است}}|=4\sqrt{3a}=2\sqrt{6}\Rightarrow 2\sqrt{3a}=\sqrt{6}\Rightarrow\sqrt{a}=\frac{\sqrt{2}}{2}\Rightarrow\boxed{a=\frac{1}{2}}$$

4

$y=-ax+2\rightarrow$ شیب $=-a$ و عرض نقطه‌ی تماس خط با ۴ $=-4a+2=2-4a$



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$$\Rightarrow f'(4)=-a,\quad f(4)=2-4a\Rightarrow f(4)+f'(4)=-1\Rightarrow 2-4a-a=-1$$

$$\Rightarrow 5a=3\Rightarrow a=\frac{3}{5}\Rightarrow\boxed{f'(4)=-\frac{3}{5}}$$

5

$$\frac{x+5}{7}=\frac{ax-1}{3x+1}\Rightarrow 3x^2+16x+5=7ax-7\Rightarrow 3x^2+(16-7a)x+12=0$$

$$\Rightarrow x^2+\left(\frac{16-7a}{3}\right)x+4=0$$

معادله برخورد دو منحنی دارای یک ریشه می‌شود:
اگر در آن نقطه مماس نیز بوده باشند
$\Delta=0$ و عبارت مربع کامل است

$\rightarrow (x+2)^2=0\Rightarrow x=-2$ غ ق ق $(x\geq 0)$

$\rightarrow (x-2)^2=0$ ق ق $\Rightarrow x^2-4x+4=0$

$$\Rightarrow\frac{16-7a}{3}=-4\Rightarrow 16-7a=-12\Rightarrow 7a=28\Rightarrow\boxed{a=4}$$

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۶

$f(x)=x^3+ax-b \;,\; y=0 \;,\; y'=0 \Rightarrow f'(x)=3x^2+a$

$\xrightarrow[x=2]{\text{مماس}} \begin{cases} f(2)=8+2a-b=0 \\ f'(2)=12+a=0 \end{cases} \Rightarrow \underbrace{f'(2)-f(2)}_{0}=4+a+b \Rightarrow \boxed{b-a=-4}$

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~~$f(x)=\dfrac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow f^{-1}(x)=\dfrac{\sqrt{x}+1}{\sqrt{x}-1}=f(x)$~~

~~برای اینکه وارون تابع هموگرافیک $y=\dfrac{ax+b}{cx+d}$ با خودش برابر باشد آنگاه $a+d=0$ ... معکوس خودش را دارند.~~

~~$f(x)=f^{-1}(x)=g(\sqrt{x}) \Rightarrow f'(x)=\dfrac{1}{2\sqrt{x}}g'(\sqrt{x})$ فرض کنید $g(x)=\dfrac{x+1}{x-1}$~~

~~$g'(x)=\dfrac{-1-1}{(x-1)^2}=-\dfrac{2}{(x-1)^2} \Rightarrow g'(\sqrt{x})=-\dfrac{2}{(\sqrt{x}-1)^2} \Rightarrow f'(x)=\dfrac{1}{2\sqrt{x}}\left(-\dfrac{2}{(\sqrt{x}-1)^2}\right)=-\dfrac{1}{\sqrt{x}(\sqrt{x}-1)^2}$~~

~~پاسخ صفحه بعد~~

~~$f'(2)=-\dfrac{1}{\sqrt{2}(\sqrt{2}-1)^2}=-\dfrac{1}{\sqrt{2}(3-2\sqrt{2})} \Rightarrow \boxed{\dfrac{1}{4-3\sqrt{2}}}$~~

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$f'(x)=2nx\sin^{n-1}(x^2)\cos x^2 \Rightarrow f(x)f'(x)=2nx\sin^{n}(x^2)\sin^{n-1}(x^2)\cos(x^2)$

$\text{حاصل} = \lim_{x\to 0}\dfrac{2nx\sin^{2n-1}(x^2)\overbrace{\cos(x^2)}^{1}}{(1-\cos x)^m}=\lim_{x\to 0}\dfrac{2nx\times(x^2)^{2n-1}}{(1-\cos x)^m}=\lim_{x\to 0}\dfrac{2nx^{4n-1}}{(1-\cos x)^m}=\lim_{x\to 0}\dfrac{2nx^{4n-1}}{(1-(1-\frac{1}{2}x^2))^m}$

$=\lim_{x\to 0}\dfrac{2nx^{4n-1}}{(\frac{1}{2})^m x^{2m}}=32\sqrt{2}$ ، $\Rightarrow 4n-1=2m \Rightarrow \boxed{m=2n-\frac{1}{2}}$

چون حاصل حد عددی شده و ... بصورت صفر بر عدد تا رفع ابهام انجام شد و حاصل عددی صفر نشد!

~~$\Rightarrow 4n=32\sqrt{2} \Rightarrow n=8\sqrt{2} \;,\; 4n-1=2m \Rightarrow 32\sqrt{2}-1=2m$~~

~~$m-n=(32\sqrt{2}-1)-(8\sqrt{2})=24\sqrt{2}$~~

$\lim_{x\to 0}\dfrac{2n\times 2^m x^{4n-1}}{x^{2m}}=2n\times 2^m=32\sqrt{2} \Rightarrow n\times 2^m=16\sqrt{2} \Rightarrow n\times 2^{2n-\frac{1}{2}}=2^{\frac{9}{2}} \Rightarrow n=2 \Rightarrow 2m=7 \Rightarrow 2m+n=7+2=9$

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$f(0)=1 \;,\; f(-1)=8-8a \;,\; \underset{[-1,0]}{\text{متوسط تغییرات}}=\dfrac{f(0)-f(-1)}{0-(-1)}=f(0)-f(-1)=1+8a-8=-11$

$\Rightarrow \boxed{a=-\frac{1}{2}} \Rightarrow f(x)=(x^2+1)^3(-\tfrac{1}{2}x+1) \Rightarrow f'(x)=3(x^2+1)^2(2x)(-\tfrac{1}{2}x+1)+(-\tfrac{1}{2})(x^2+1)^3$

$\Rightarrow x=-2a=1 \rightsquigarrow f'(1)=3\times 2^2\times 2\times\tfrac{1}{2}-\tfrac{1}{2}\times 2^3=12-4=\boxed{8}$

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$f(x) = \sin x \cos 2x \Rightarrow f(\frac{\pi}{4}) = 0 ,\ f(\frac{\pi}{2}) = -1 \Rightarrow$ آهنگ تغییرات $f = \dfrac{f(\frac{\pi}{2}) - f(\frac{\pi}{4})}{\frac{\pi}{2} - \frac{\pi}{4}} = \dfrac{-1-0}{\frac{\pi}{4}} = -\dfrac{4}{\pi}$ ۱۵

$g(x) = \sin^4 x - \cos^4 x = \underbrace{(\sin^2 x + \cos^2 x)}_{1}\underbrace{(\sin^2 x - \cos^2 x)}_{-\cos 2x} = -\cos 2x \Rightarrow g(\frac{\pi}{2}) = 1 ,\ g(\frac{\pi}{4}) = 0$ ۲

آهنگ متوسط تغییرات $g = \dfrac{g(\frac{\pi}{2}) - g(\frac{\pi}{4})}{\frac{\pi}{2} - \frac{\pi}{4}} = \dfrac{1-0}{\frac{\pi}{4}} = \dfrac{4}{\pi} \Rightarrow \boxed{\dfrac{\text{آهنگ تغییرات } f}{\text{آهنگ تغییرات } g} = \dfrac{-\frac{4}{\pi}}{\frac{4}{\pi}} = -1}$

$y = \dfrac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow y\sqrt{x} - y = \sqrt{x}+1 \Rightarrow (y-1)\sqrt{x} = y+1 \Rightarrow \sqrt{x} = \dfrac{y+1}{y-1} \Rightarrow x = \left(\dfrac{y+1}{y-1}\right)^2$ ۸

$\Rightarrow f^{-1}(x) = \left(\dfrac{x+1}{x-1}\right)^2 \Rightarrow (f^{-1})'(x) = 2\left(\dfrac{x+1}{x-1}\right)\left(\dfrac{-2}{(x-1)^2}\right) \Rightarrow (f^{-1})'(2) = 2\left(\dfrac{3}{1}\right)\left(\dfrac{-2}{1^2}\right) = \boxed{-12}$ ۲