آرمین نوری ۱۸

$$g'(x) = \left(2\tan x \times (1+\tan^2 x) - \sqrt{2}\sin x\right) \times f'(\tan^2 x + \sqrt{2}\cos x)$$

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$$g'(\pi/4) = (2\times 2 - 1) \times f'(2) = \sqrt{3} \to 3f'(2) = \sqrt{3} \qquad f'(2) = \sqrt{3}/3$$

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(۲

$$f(x) - 2g(x) = \frac{(2+\cos x)(4+\cos^2 x - 4\cos x)}{(2-\cos x)(2+\cos x)} - \frac{4}{2-\cos x} = \frac{\cos^2 x - 2\cos x}{2-\cos x}$$

$$\frac{-\cos x(2-\cos x)}{2-\cos x} = -\cos x \qquad f'(x) - 2g'(x) = \sin x \qquad \sin(7\pi/6) = -1/2$$

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(۳

$$f(3/4^+) = (4x-3)\sqrt{3/4\, a} \qquad f'_+(3/4) = 4\sqrt{3/4\, a} + 0 = 2\sqrt{3a}$$

$$f(3/4^-) = (3-4x)\sqrt{3/4\, a} \qquad f'_-(3/4) = -4\sqrt{3/4\, a} + 0 = -2\sqrt{3a}$$

$$4\sqrt{3a} = 2\sqrt{6} \qquad 2\sqrt{3a} = \sqrt{6} \to \sqrt{12a} = \sqrt{6} \qquad a = 1/2$$

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$$\left.\begin{array}{l} y = -ax + 2 \to f(4) = -4a + 2 \\ f'(x) = -a \to f'(4) = -a \end{array}\right\} \to f(4) + f'(4) = -5a + 2 = -1 \to a = 3/5$$

$$-a \to -3/5$$

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$$\cancel{7y = x + 5 \to y = \tfrac{1}{7}x + \tfrac{5}{7} \to y' = \tfrac{1}{7}}$$

$$\cancel{y = \frac{ax-1}{3x+1} \to y' = \frac{a(3x+1) - 3(ax-1)}{(3x+1)^2} = \frac{3ax + a - 3ax + 3}{9x^2 + 1 + 6x} = \frac{a+3}{9x^2+1+6x} = \frac{1}{7}}$$

$$7a + 21 = 9x^2 + 1 + 6x \to 9x^2 + 6x - 20 - 7a = 0$$

$$\left.\begin{array}{l} y = \dfrac{x+5}{7} \\ y = \dfrac{ax-1}{3x+1} \end{array}\right\} \to \frac{x+5}{7} = \frac{ax-1}{3x+1} \to 3x^2 + 16x + 5 = 7ax - 7$$

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$$3x^2 + (16 - 7a)x + 12 = 0$$

$$\Delta = 0 \to (16-7a)^2 - 4\times 36 = 0 \to (16-7a)^2 = 144 \to \begin{cases} a = 4/7 \text{ غقق} \\ a = 4 \end{cases}$$

آرمین نوری (۶

$$f'(x)=3x^2+a \Rightarrow f'(2)=12+a=0 \rightarrow a=-12$$

$$f(x)=x^3-12x-b \Rightarrow f(2)=8-24-b=0 \qquad b=-16$$

$$b-a=-16+12=\underline{-4}$$

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$$\lim_{x\to 0}\frac{\sin^n(x^2)\times n\sin^{n-1}(x^2)\times \cos(x^2)\times(2x)}{\left(2\sin^2\frac{x}{2}\right)^m} \xrightarrow{\text{هم‌ارزی}} \lim_{x\to 0}\frac{(x^2)^n n(x^2)^{n-1}\times 2x}{\left(2\left(\frac{x}{2}\right)^2\right)^m}=$$

$$\lim_{x\to 0}\frac{2nx^{2n+2n-2+1}}{\left(\frac{x^2}{2}\right)^m}=\lim_{x\to 0}\frac{2nx^{4n-1}}{\frac{1}{2^m}x^{2m}}=\lim_{x\to 0}2^{m+1}n\,x^{4n-2m-1}$$ حاصل حد عددی متناهی غیرصفر ← توان صفر

$$\begin{cases}4n-2m-1=0 \longrightarrow n=\frac{2m+1}{4}\\ 2^{m+1}n=32\sqrt{2}\end{cases} \longrightarrow 2^{m+1}\times\frac{2m+1}{4}=32\sqrt{2}\rightarrow(2m+1)2^{m+1}=2^7\sqrt{2}\longrightarrow m=\frac{7}{2},\ n=2$$

$$2m+n=9$$

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$$y=\frac{\sqrt{x}+1}{\sqrt{x}-1} \qquad \sqrt{x}\,y-y=\sqrt{x}+1 \qquad \sqrt{x}\,y-\sqrt{x}=y+1 \qquad \sqrt{x}=\frac{y+1}{y-1}$$

$$f^{-1}(x)=\left(\frac{x+1}{x-1}\right)^2 \qquad f^{-1\prime}(x)=2\left(\frac{x+1}{x-1}\right)\left(\frac{(x-1)-(x+1)}{(x-1)^2}\right)$$

$$f^{-1\prime}(2)=2\times 3\times -2=\underline{-12}$$

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$$\frac{f(0)-f(-1)}{0+1}=8a-7=-11 \rightarrow a=-\frac{1}{2}$$

$$f(x)=(x^2+1)^3\left(-\frac{x}{2}+1\right)$$

$$f'(x)=3(x^2+1)^2(2x)\left(-\frac{x}{2}+1\right)+-\frac{1}{2}(x^2+1)^3$$

$$f'(x)\Rightarrow f'(1)=24\times\frac{1}{2}-4=\underline{8}$$

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$$y=\sin x\cos 2x \longrightarrow \frac{f(\pi/2)-f(\pi/4)}{\pi/2-\pi/4}=\frac{-1}{\pi/4}=\frac{-4}{\pi}$$

$$y=\sin^4-\cos^4 \longrightarrow \frac{f(\pi/2)-f(\pi/4)}{\pi/2-\pi/4}=\frac{1}{\pi/4}=\frac{4}{\pi}$$

$$\frac{-4/\pi}{\frac{4}{\pi}}=\underline{-1}$$

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