حسین شهبازی

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$g(x) = f(\tan^2 x + \sqrt{2}\cos x)$

$g'(x) = \left(2(1+\tan^2 x)(\sqrt{2}\cos x) + -\sqrt{2}\sin x(\tan^2 x)\right) f'(\tan^2 x + \sqrt{2}\cos x)$

$g'(\frac{\pi}{4}) = \left(\underbrace{2(1+1)\cancel{(\sqrt{2}\times\frac{\sqrt{2}}{2})}}_{4} + \underbrace{-\sqrt{2}\times\frac{\sqrt{2}}{2}}_{-1}(1)\right) f'\left(1+\cancel{\frac{\sqrt{2}\times\sqrt{2}}{2}}\right) \Rightarrow 3f'(2) = \sqrt{3}$

$\boxed{f'(2) = \frac{\sqrt{3}}{3}}$

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$f(x) = |4x-3|\sqrt{ax}$

$f'_+(\frac{3}{4}) = \lim_{x\to\frac{3}{4}^+} \frac{(4x-3)\sqrt{ax} - 0}{x-\frac{3}{4}} \xrightarrow{hop} (4)\sqrt{ax} + \frac{(4x-3)(a)}{2\sqrt{ax}}$

$f'_+(x) - f'_-(x) = 2\sqrt{6}$

$f'_-(\frac{3}{4}) = \lim_{x\to\frac{3}{4}^-} = \frac{-(4x-3)\sqrt{ax} - 0}{x-\frac{3}{4}} = \frac{0}{0} \xrightarrow{hop} (-4)(\sqrt{ax}) + \frac{(-4x+3)(a)}{2\sqrt{ax}}$

$4\sqrt{ax} + \frac{\cancel{(4x-3)}a}{2\sqrt{ax}} - (-4\sqrt{ax}) + \frac{\cancel{(-4x+3)}(a)}{2\sqrt{ax}} = 8\sqrt{ax} = 2\sqrt{6}$ $\quad 4\sqrt{\frac{3a}{4}} = \sqrt{6} = \sqrt{12a} = \sqrt{6}$

$\boxed{a = 1/2}$

$y + ax - 2 \xrightarrow{x=4} y = 2 - ax \quad y_4 = 2 - 4a$

$f(4) + f'(4) = 1$ $\quad 2 - 4a + (-a) = 1 \quad 2 - 5a = 1$

$f(4) = 2 - 4a$ $\quad -5a = -1$

$\boxed{a = 1/5}$

$f'(4) = -a$ $\quad \boxed{f'(4) = -1/5}$

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$7y - x = 5$

$7y = 5 + x \quad y = \frac{5+x}{7}$

$\frac{ax-1}{3x+1} = \frac{5+x}{7} \quad 7ax - 7 = 3x^2 + 16x + 5$

$3x^2 + (16 - 7a) + 12 = 0$

$3(x-2)^2$

$\frac{16-7a}{3} = -4 \quad 16 - 7a = -12$

$-7a = -28 \quad \boxed{a = 4}$

۶-

$f(2) = 0 \quad x^3 + ax - b \quad x^3 - 12x - b = 0 \quad 8 - 24 - b = 0 \quad \boxed{b = -16}$

$f'(2) = 0 \quad 3x^2 + a \longrightarrow 12 + a = 0 \quad \boxed{a = -12}$

$\boxed{-16 - (-12) = -4}$

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$f(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1} \quad y = \frac{\sqrt{x}+1}{\sqrt{x}-1} \quad y\sqrt{x} - y = \sqrt{x} + 1$

$y\sqrt{x} - \sqrt{x} = y + 1 \quad \sqrt{x}(y-1) = y+1 \quad \sqrt{x} = \frac{y+1}{y-1}$

$x = \left(\frac{x+1}{x-1}\right)^2$

$f^{-1}(x) = \left(\frac{x+1}{x-1}\right)^2$

$f^{-1\prime}(x) = 2\left(\frac{x+1}{x-1}\right)\left(\frac{-2}{(x-1)^2}\right) = 2(3)(-2) = \boxed{-12}$

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$$f(x) = (x^2+1)^3(ax+1)$$

$$\frac{f(0)-f(-1)}{0-(-1)} = \frac{1-f(-1)}{1} = -11 \qquad -f(-1) = -12$$

$$f(-1) = 12 \longrightarrow \underbrace{\cancel{(1+1)^3}}_{8}(-a+1) = -12$$

$$-8a+8 = -12$$

$$-8a = -20 \qquad a = \frac{20}{8} \qquad \boxed{a = \frac{5}{2}}$$

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$$\frac{f(b)-f(a)}{b-a}$$

$$f\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2}\cos\pi = -1$$

$$f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4}\cancel{\cos\frac{\pi}{2}} = 0 \qquad \frac{-1-0}{\frac{\pi}{2}-\frac{\pi}{4}} = \frac{-1}{\frac{\pi}{4}} = \frac{-4}{\pi}$$

$$g(x) = \sin^4 x - \cos^4 x \Longrightarrow \cos 2x$$

$$\frac{g(b)-g(a)}{b-a} \qquad g\left(\frac{\pi}{2}\right) = \cos\pi = -1$$

$$g\left(\frac{\pi}{4}\right) = \cos\frac{\pi}{2} = 0 \qquad \frac{-1-0}{\frac{\pi}{2}-\frac{\pi}{4}} = \frac{-4}{\pi} \qquad \boxed{\frac{\Delta f}{\Delta g} = \frac{\cancel{-4}/\cancel{\pi}}{\cancel{-4}/\cancel{\pi}} = 1}$$