۱- حسین شهبازی ۱۳

$g(x) = f(\tan^2 x + \sqrt{2}\cos x)$

$g'(x) = \left(2(1+\tan^2 x)(\tan x) - \sqrt{2}\sin x\right) f'(\tan^2 x + \sqrt{2}\cos x)$

$g'(\frac{\pi}{4}) = \left(2(1+1)(1) - \sqrt{2}\times\frac{\sqrt{2}}{2}\right) f'(1 + \sqrt{2}\times\frac{\sqrt{2}}{2}) \Rightarrow 3f'(2) = \sqrt{2}$

$\boxed{f'(2) = \frac{\sqrt{2}}{3}}$

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سوال ۲

$f'(\frac{V\pi}{4}) - 2g'(\frac{V\pi}{4}) = (f-2g)'(\frac{V\pi}{4})$

$\rightarrow (f-2g)' = \left(\frac{8+\cos^3 x}{4-\cos^2 x} - \frac{4}{2-\cos x}\right) = \left(\frac{(\cos x+2)(\cos^2 x - 2\cos x + 4)}{(2-\cos x)(2+\cos x)} - \frac{4}{2-\cos x}\right) =$

$\left(\frac{\cos(\cos x - 2)}{-(\cos x - 2)}\right)' = -(\cos x)' = \sin x \xrightarrow{x = V\pi/4} \sin(\frac{V\pi}{4}) = \frac{-1}{2}$

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$f(x) = |4x-3|\sqrt{ax}$

$f'_+(\frac{3}{4}) = \lim_{x\to\frac{3}{4}^+} \frac{(4x-3)\sqrt{ax} - 0}{x - \frac{3}{4}} \xrightarrow{hop} (4)\sqrt{ax} + \frac{(4x-3)(a)}{2\sqrt{ax}}$

$f'_+(x) - f'_-(x) = 2\sqrt{6}$

$f'_-(\frac{3}{4}) = \lim_{x\to\frac{3}{4}^-} \frac{-(4x-3)\sqrt{ax} - 0}{x - \frac{3}{4}} = \frac{0}{0} \xrightarrow{hop} (-4)(\sqrt{ax}) + \frac{(-4x+3)(a)}{2\sqrt{ax}}$

$4\sqrt{ax} + \frac{(4x-3)a}{2\sqrt{ax}} - (-4\sqrt{ax}) + \frac{(-4x+3)(a)}{2\sqrt{ax}} = 8\sqrt{ax} = 2\sqrt{6} \quad 4\sqrt{\frac{3a}{4}} = \sqrt{6} \quad = \sqrt{12a} = \sqrt{6}$

$\boxed{a = 1/2}$

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$y + ax = 2 \xrightarrow{x=4} y = 2 - ax \quad y_4 = 2 - 4a$

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$f(4) + f'(4) = 1 \quad 2 - 4a + (-a) = 1 \quad 2 - 5a = 1$

$f(4) = 2 - 4a \quad -5a = -1$

$f'(4) = -a \quad \boxed{a = 1/5} \quad \boxed{f'(4) = -1/5}$

$f(4) + f'(4) = -4a + 2 - a = -1 \rightarrow a = \frac{3}{5}$

$\rightarrow f'(4) = -a = -\frac{3}{5} = -0.6$

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$$7y - x = 5$$
$$7y = 5 + x \qquad y = \frac{5+x}{7}$$

$$\frac{ax-1}{3x+1} = \frac{5+x}{7} \qquad 7ax - 7 = 3x^2 + 16x + 5$$

$$3x^2 + (16 - 7a) + 12 = 0$$
$$3(x-2)^2$$
$$\frac{16-7a}{3} = -4 \qquad 16 - 7a = -12$$
$$-7a = -28 \quad \boxed{a = 4}$$

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$$f(2) = 0 \qquad x^3 + ax - b \qquad x^3 - 12x - b = 0 \qquad 8 - 24 - b = 0 \quad \boxed{b = -16}$$
$$f'(2) = 0 \qquad 3x^2 + a \longrightarrow 12 + a = 0 \quad \boxed{a = -12}$$
$$\boxed{-16 - (-12) = -4}$$

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هوپیتال

$$\lim_{x\to 0} \frac{\sin^n(x^2) \times n \sin^{n-1}(x^2) \times \cos(x^2) \times (2x)}{(2\sin^2\frac{x}{2})^m} \xrightarrow{\text{هم‌ارزی}} \lim_{x\to 0} \frac{(x^2)^n\, n\, (x^2)^{n-1} \times 2x}{(2(\frac{x}{2})^2)^m} =$$

$$\lim_{x\to 0} \frac{2nx^{2n+2n-2+1}}{(\frac{x^2}{2})^m} = \lim_{x\to 0} \frac{2nx^{4n-1}}{\frac{1}{2^m} x^{2m}} = \lim_{x\to 0} 2^{m+1} n\, x^{4n-2m-1} \rightsquigarrow$$ حاصل حد عدد غیر صفر متناهی شده پس توان صفر میشود

$$\begin{cases} 4n - 2m - 1 = 0 \longrightarrow n = \frac{2m+1}{4} \\ 2^{m+1} n = 32\sqrt{2} \end{cases} \longrightarrow 2^{m+1} \times \frac{2m+1}{4} = 32\sqrt{2} \to (2m+1)2^{m+1} = 2^7\sqrt{2} \to m = \frac{7}{2}$$
$$n = 2$$
$$2m + n = 9$$

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$$f(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1} \qquad y = \frac{\sqrt{x}+1}{\sqrt{x}-1} \qquad y\sqrt{x} - y = \sqrt{x} + 1$$
$$y\sqrt{x} - \sqrt{x} = y + 1 \qquad \sqrt{x} = \frac{y+1}{y-1}$$
$$\sqrt{x}(y-1) = y + 1$$
$$f^{-1}(x) = \left(\frac{x+1}{x-1}\right)^2 \qquad y = \left(\frac{x+1}{x-1}\right)^2$$

$$f^{-1\prime}(x) = 2\left(\frac{x+1}{x-1}\right)\left(\frac{-2}{(x-1)^2}\right) = 2(3)(-2) = \boxed{-12}$$

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9

1

$$f(x) = (x^2+1)^3(ax+1)$$

$$\frac{f(0)-f(-1)}{0-(-1)} = \frac{1-f(-1)}{1} = -11 \qquad -f(-1) = -12$$

$$f(-1) = -12 \longrightarrow \underbrace{\cancel{(1+1)^3}}_{8}(-a+1) = -12$$

$$-8a+8 = -12$$

$$-8a = -20 \qquad a = \frac{20}{8} \qquad \boxed{a = \frac{5}{2}}$$

$$8a-7 = -11 \longrightarrow \begin{cases} a = \dfrac{-1}{2} \\ x = 1 \end{cases}$$

$$f'(x) = 3(2x)(x^2+1)^2(ax+1) + a(x^2+1)^3 \xrightarrow[x=\frac{-1}{2}]{x=1}$$

$$3(2\times 1)(1+1)^2\left(\frac{-1}{2}\times 1+1\right) + \frac{-1}{2}(1+1)^3 = 8$$

-10

$$\frac{f(b)-f(a)}{b-a}$$

$$y_2 = \sin^4 x - \cos^4 x = -\cos 2x$$

$$\begin{cases} y_2\left(\frac{\pi}{4}\right) = -\cos\left(\frac{\pi}{2}\right) = 0 \\ y_2\left(\frac{\pi}{2}\right) = -\cos(\pi) = 1 \end{cases} \longrightarrow \frac{\Delta y_2}{\Delta x} = \frac{1-0}{\frac{\pi}{2}-\frac{\pi}{4}} = \frac{4}{\pi}$$

$$f\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2}\cos\pi = -1$$

$$f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4}\cancel{\cos\frac{\pi}{2}} = 0 \qquad \frac{-1-0}{\frac{\pi}{2}-\frac{\pi}{4}} = \frac{-1}{\frac{\pi}{4}} = -\frac{4}{\pi}$$

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$$\longrightarrow \frac{\frac{\Delta y_1}{\Delta x}}{\frac{\Delta y_2}{\Delta x}} = \frac{-\frac{4}{\pi}}{\frac{4}{\pi}} = -1$$

$$g(x) = \sin^4 x - \cos^4 x \Longrightarrow \cos 2x$$

$$\frac{g(b)-g(a)}{b-a} \qquad g\left(\frac{\pi}{2}\right) = \cos\pi = -1 \qquad g\left(\frac{\pi}{4}\right) = \cos\frac{\pi}{2} = 0$$

$$\frac{-1-0}{\frac{\pi}{2}-\frac{\pi}{4}} = \frac{-4}{\pi} \qquad \boxed{\frac{\Delta f}{\Delta g} = \frac{\cancel{-4/\pi}}{\cancel{-4/\pi}} = 1}$$