

$$g(r) = f(\tan r + \sqrt{r} \cos r), \quad g'(\frac{\pi}{4}) = \sqrt{3}$$

$$g(\frac{\pi}{4}) = f(1 + \sqrt{\frac{\pi}{4}} \cdot \frac{\sqrt{2}}{2})$$

$$f'(r) = \frac{\sqrt{r}}{r} \quad \checkmark$$

$$\Rightarrow g'(\frac{\pi}{4}) = f'(r) \cdot (2 \tan r \sec^2 r - \sqrt{r} \sin r) \Rightarrow g'(\frac{\pi}{4}) = f'(1) \cdot \sqrt{3} \quad (2)$$

$$f(x) = \frac{1 + \cos^3 x}{1 - \cos^3 x} \Rightarrow f'(x) = \frac{(-3 \cos^2 x \cdot \sin x)(1 - \cos^3 x) - (1 + \cos^3 x)(3 \cos^2 x \sin x)}{(1 - \cos^3 x)^2}$$

$$g(x) = \frac{2}{2 - \cos x} \Rightarrow g(x) = 2(2 - \cos x)^{-1} \Rightarrow g'(x) = 2(-1)(2 - \cos x)^{-2} \cdot \sin x \quad (1.5)$$

$$\text{حاصل‌خیز} = \frac{3\sqrt{3}}{2} - \sqrt{3} = \frac{\sqrt{3}}{2} \quad (\text{جواب پایین صفت})$$

این ۲ مقدار برابرند پس $\Rightarrow 2f'(\frac{3}{4}) = 2\sqrt{4} \Rightarrow f'(\frac{3}{4}) = \sqrt{4}$

$$f'_+(\frac{3}{4}), f'_-(\frac{3}{4}) \rightarrow \left((x-2)\sqrt{ax} \right)' \Rightarrow (x-2)\sqrt{ax} \rightarrow \sqrt{ax} + (x-2) \cdot \frac{a}{2\sqrt{ax}}$$

$$\Rightarrow \frac{16ax + 16ax - 12a}{2\sqrt{ax}} = \sqrt{4} \Rightarrow \frac{32ax - 12a}{2\sqrt{ax}} = 2 \Rightarrow 32ax - 12a = 4\sqrt{ax} \Rightarrow 32a^2 = 16a \Rightarrow a = \frac{1}{4}$$

$$y = -ax + 2 \rightarrow y' = -a, \quad f(1) = -1a + 2$$

$$\left. \begin{aligned} f'(1) + f(1) &= -1 \\ -1a + 2 - a &= -1 \end{aligned} \right\} \Rightarrow a = \frac{3}{2}, \quad f'(1) = -\frac{3}{2} \quad \checkmark \quad (2)$$

$$y = x + 5 \Rightarrow y = \frac{x+5}{v} \quad \left\{ \begin{aligned} \frac{x+5}{v} &= \frac{ax-1}{3x+1} \Rightarrow 3x^2 + 14x + 5 = vx - v \\ 3x^2 + (14-v)x + 11 &= 0 \end{aligned} \right.$$

$$\Rightarrow 3(x-2)^2 \Rightarrow 14-v = -12 \Rightarrow v = 26$$

$$\boxed{a=4} \quad \checkmark \quad (2)$$

$$f(x) = x^3 + ax - b \Rightarrow f'(x) = 3x^2 + a \Rightarrow 0 = 3x^2 + a \Rightarrow a = -12$$

$$x^3 - 12x - b \Rightarrow x^2 - 12x - b = 0 \Rightarrow b = -14$$

$$b - a = -14 - (-12) = -2$$

$$f(x) = \sin^n(x^m) \sim x^{mn} \quad f'(x) \sim mn x^{mn-1} \Rightarrow f(x) \cdot f'(x) = \sin^n(x^m) \cdot mn x^{mn-1}$$

$$(1 - \cos x)^m \sim \left(\frac{x^2}{2}\right)^m = \frac{x^{2m}}{2^m} \Rightarrow \frac{2^m x^{2m-1}}{2^m} = 2^m x^{2m-1} = 2^m x^{2m-1}$$

$$2^m x^{2m-1} = 2^m x^{2m-1} \Rightarrow 2^m x^{2m-1} = 2^m x^{2m-1} \Rightarrow 2^m x^{2m-1} = 2^m x^{2m-1}$$

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$$f(x) = \frac{\sqrt{x+1}}{\sqrt{x-1}} \Rightarrow f^{-1}(x) \Rightarrow y\sqrt{x}-y = \sqrt{x+1} \Rightarrow y\sqrt{x}-\sqrt{x} = y+1$$

$$\Rightarrow \sqrt{x}(y-1) = y+1 \Rightarrow \sqrt{x} = \frac{y+1}{y-1}$$

$$x = \left(\frac{y+1}{y-1}\right)^2 \Rightarrow x = \frac{y^2+2y+1}{y^2-2y+1} \Rightarrow x = \frac{y^2+2y+1}{y^2-2y+1}$$

$$\Rightarrow \text{Jawab} = -12$$

$$\frac{f(b) - f(a)}{b - a} = \frac{1 + \ln(a-1)}{a - (-1)} = -11 \Rightarrow \ln(a-1) = -12$$

$$a-1 = e^{-12} \Rightarrow a = 1 + e^{-12}$$

$$f(0) = 1, f(-1) = \ln(-a+1) \Rightarrow a = 1$$

$$f'(x) = 2(x)(x^2+1)^{-1/2}(2x+1) + a(x^2+1)^{-3/2} \xrightarrow{x=1} 2(1)(1+1)^{-1/2}(2+1) + \frac{1}{2}(1+1)^{-3/2} = 1$$

$$y_1 = \frac{f(\frac{\pi}{4}) - f(\frac{\pi}{2})}{\frac{\pi}{4} - \frac{\pi}{2}} = \frac{-1}{-\frac{\pi}{4}} = \frac{4}{\pi}$$

$$y_2 = \frac{f(\frac{\pi}{4}) - f(\frac{\pi}{6})}{\frac{\pi}{4} - \frac{\pi}{6}} = \frac{1}{\frac{\pi}{12}} = \frac{12}{\pi}$$

$$\Rightarrow \text{Jawab} = \frac{4}{\pi} - \frac{12}{\pi} = -\frac{8}{\pi}$$

$$f'(\frac{\sqrt{r}}{r}) - rg'(\frac{\sqrt{r}}{r}) = (f - rg)'(\frac{\sqrt{r}}{r})$$

$$\rightarrow (f - rg)' = \left(\frac{1 + \cos^2 u}{r - \cos^2 u} - \frac{r}{r - \cos^2 u} \right) = \left(\frac{(\cos^2 u + 1)(\cos^2 u - r \cos^2 u + r)}{(r - \cos^2 u)(r + \cos^2 u)} - \frac{r}{r - \cos^2 u} \right) =$$

$$\left(\frac{\cos(\cos^2 u - r)}{-\cos^2 u - r} \right)' = -(\cos^2 u)' = \sin u \xrightarrow{u = \frac{\sqrt{r}}{r}} \sin\left(\frac{\sqrt{r}}{r}\right) = \frac{-1}{r}$$