محمدعلی جعفری — دوازدهم تجربی A

۱۷٫۵

۱-

$g'(x)=(2\tan x-\sqrt{3}\sin x)f'(\tan^2 x+\sqrt{3}\cos x)$

$g'(\frac{\pi}{6})=(2-(2\sqrt{3})(\frac{\sqrt{3}}{2}))f'(1+\frac{(\sqrt{3})(\sqrt{3})}{2})=\sqrt{3}$

$\Rightarrow \boxed{f'(2)=\sqrt{3}}$

[۱٫۵]

$x=\frac{\pi}{4} \rightarrow g'(\frac{\pi}{4})=3f'(2)=\sqrt{3} \rightarrow f'(2)=\frac{1}{\sqrt{3}} \rightarrow f'(2)=\frac{\sqrt{3}}{3}$

۲-

$f(x)-2g(x) \Rightarrow \frac{8+\sin^3 x}{4-\sin^2 x}-\frac{(2+\sin x)(4+\sin^2 x-2\sin x)}{2-\sin x}$

$(2-\sin x)(2+\sin x)$

$\Rightarrow f(x)-2g(x)=\frac{\sin^2 x-2\sin x}{2-\sin x}=-\sin x$

$\Rightarrow h(x)=-\cos x$ پس $h'(x)=\sin x$

$\Rightarrow h'(\frac{7\pi}{6})=\sin(\frac{7\pi}{6})=\boxed{-\frac{1}{2}}$ ✓

[۲]

۳-

$if\ x>\frac{3}{4} \Rightarrow f(x)=(4x-3)\sqrt{ax}$

$f'(x)=4\sqrt{ax} \Rightarrow f'(\frac{3}{4})=4\sqrt{\frac{3a}{4}}=2\sqrt{3a}$

$if\ x<\frac{3}{4} \Rightarrow f(x)=(3-4x)\sqrt{ax}$

$f'(x)=-4\sqrt{ax} \Rightarrow f'(\frac{3}{4})=-2\sqrt{3a}$

$\Rightarrow 2\sqrt{3a}-(-2\sqrt{3a})=2\sqrt{6} \Rightarrow 4\sqrt{3a}=2\sqrt{6}$

$\Rightarrow (16)(3a)=(4)(6) \Rightarrow \boxed{a=\frac{1}{2}}$ ✓

[۲]

۴-

$y=-ax+2 \Rightarrow f'(4)=-a$

$f(4)=-4a+2$

$f(4)+f'(4)=-4a+2-a=-1$

$\Rightarrow a=\frac{3}{5} \Rightarrow \boxed{f'(4)=-\frac{3}{5}}$ ✓

[۲]

۵-

$\frac{ax-1}{3x+1}=\frac{x+5}{7} \Rightarrow 3x^2+16x+5=7ax-7$

$\Rightarrow 3x^2+x(16-7a)+12=0$

$\Rightarrow x^2+\frac{x(16-7a)}{3}+4=0$

$(x-2)^2=x^2-4x+4$

$\Rightarrow \frac{16-7a}{3}=-4 \Rightarrow 16-7a=-12$

$\boxed{a=4}$ ✓

[۲]

۶-

$f(x)=x^3+ax-b$ پس $f'(x)=3x^2+a$

$f'(2)=0 \Rightarrow 12+a=0 \rightarrow a=-12$

$f(2)=0 \Rightarrow 8+2a-b=0 \Rightarrow 8+2(-12)-b=0$

$\Rightarrow b=-16$

$b-a=-16-(-12)=\boxed{-4}$ ✓

[۲]

۷-

$f(x)=\sin^n(x^2) \Rightarrow f'(x)=2n\sin^{n-1}(x^2)\,2x\cos(x^2)$

$\lim \frac{f(x)f'(x)}{(1-\sin)^m}=\frac{11}{4}$

$(1-\sin)^m=2 \Rightarrow \sin x=-1$

[۰٫۵]

(جواب در پایین صفحه)

۸-

$f^{-1}(x): x=\frac{\sqrt{y}+1}{\sqrt{y}-1} \Rightarrow f^{-1}(x)=(\frac{x+1}{x-1})^2$

$(f^{-1})'(x)=2(\frac{x+1}{x-1})(\frac{-2}{(x-1)^2})$; $x=2$: $(2)(\frac{3}{1})(\frac{-2}{1})$

$(2)(3)(-2)=-12$

[۱٫۵]

۹-

$f(x)=(x^2+1)^3(ax+1) \Rightarrow f(0)=1$

$f(-1)=8-8a$

$\Rightarrow \frac{f(0)-f(-1)}{0-(-1)}=-11 \Rightarrow \frac{1+8a-8}{1}=-11$

$\Rightarrow a=-\frac{1}{2} \Rightarrow f'(x)=3(x^2+1)^2(2x)(-\frac{1}{2}x+1)+(-\frac{1}{2})(x^2+1)^3$

$\Rightarrow f'(1)=(3)(4)(2)(\frac{1}{2})+(-\frac{1}{2})(8)=\boxed{8}$ ✓

[۲]

ب -

$$\frac{\frac{f(\frac{\pi}{2}) - f(\frac{\pi}{3})}{\frac{\pi}{2} - \frac{\pi}{3}}}{\frac{g(\frac{\pi}{2}) - g(\frac{\pi}{3})}{\frac{\pi}{2} - \frac{\pi}{3}}} \Rightarrow \frac{\sin^2\frac{\pi}{2}\cos\pi - \sin^2\frac{\pi}{3}\cos\frac{\pi}{2}}{(\sin^3\frac{\pi}{2} - \cos^2\frac{\pi}{2}) - (\sin^3\frac{\pi}{3} - \cos^3\frac{\pi}{3})} \quad (۲)$$

$$= \frac{-1}{1} = \boxed{-1}$$

سوال ۱۴

$f(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1} \rightarrow \sqrt{x}y - y = \sqrt{x}+1 \longrightarrow \sqrt{x}(y-1) = y+1 \rightarrow \sqrt{x} = \frac{y+1}{y-1} \rightarrow x = \left(\frac{y+1}{y-1}\right)^2 \longrightarrow y = \left(\frac{x+1}{x-1}\right)^2$

$f^{-1}(x) = \left(\frac{x+1}{x-1}\right)^2 \rightarrow (f^{-1})' = 2\left(\frac{x+1}{x-1}\right) \times \frac{-2}{(x-1)^2} \longrightarrow (f^{-1}(2))' = 2 \times 3 \times (-2) = -12$