

$$g(x) = \left(\sqrt{x} \tan x \cdot \frac{1}{\cos^2 x} - \sqrt{x} \sin x \right) \cdot f'(x) \Rightarrow x = \frac{\pi}{4}$$

$$\Rightarrow g'\left(\frac{\pi}{4}\right) = (2 \times \sqrt{x} - 1) \cdot f'(x) = \sqrt{2} \Rightarrow f'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \quad \checkmark$$

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$$f(x) = \frac{(x + \cos x)(\cos^2 x - 2 \cos x + 1)}{(x - \cos x)(x + \cos x)} = \frac{\cos^2 x - 2 \cos x + 1}{x - \cos x}$$

$$\frac{f(x) - 2g(x)}{h'(x)} = \frac{\cos^2 x - 2 \cos x + 1 - x}{x - \cos x} = \frac{\cos^2 x - 2 \cos x}{x - \cos x} = -\cos$$

$$h'(x) = \sin x \Rightarrow x = \frac{\pi}{2} \Rightarrow \sin \frac{\pi}{2} = \boxed{\frac{1}{2}} \quad \checkmark$$

$$\left. \begin{array}{l} x > \frac{\pi}{4} \Rightarrow (x-2)\sqrt{ax} \Rightarrow x\sqrt{ax} \\ x < \frac{\pi}{4} \Rightarrow -(x-2)\sqrt{ax} \Rightarrow -x\sqrt{ax} \end{array} \right\} x = \frac{\pi}{4} \Rightarrow \left. \begin{array}{l} \frac{x\sqrt{ax}}{\sqrt{x}} = x\sqrt{a} = m_1 \\ -\frac{x\sqrt{ax}}{\sqrt{x}} = -x\sqrt{a} = m_2 \end{array} \right\}$$

$$\Rightarrow |m_2 - m_1| = 2\sqrt{a} \Rightarrow x\sqrt{a} = x\sqrt{4} \Rightarrow \boxed{a = \frac{1}{4}} \quad \checkmark$$

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$$y = -ax + 2 \Rightarrow \left. \begin{array}{l} f'(x) = -a \\ f(x) = -ax + 2 \end{array} \right\} f(x) + f'(x) = -ax + 2 = -1$$

$$\Rightarrow -ax + 2 = -1 \Rightarrow a = \frac{3}{x} \Rightarrow \boxed{f'(x) = -\frac{3}{x}} \quad \checkmark$$

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$$\frac{x+6}{x} = \frac{ax-1}{x+1} \Rightarrow x^2 + 14x + 6 = ax^2 - 1 \Rightarrow x^2 + (14-a)x + 7 = 0$$

$$\Rightarrow x^2 + \frac{14-a}{x}x + 7 = 0 \Rightarrow \frac{14-a}{x} = -7 \Rightarrow 14-a = -12 \Rightarrow a = 26$$

$$\boxed{a = 26} \quad \checkmark$$

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$$f'(x) = x^2 + a \Rightarrow f'(1) = 0 \Rightarrow 1 + a = 0 \Rightarrow a = -1 \Rightarrow f(x) = x^2 - 1x - b$$

$$\Rightarrow f(1) = 0 \Rightarrow 1 - 1 - b = 0 \Rightarrow b = -1$$

$$\Rightarrow b - a = -1 + 1 = 0 \quad \checkmark$$

(2)

$$f(x) = \ln x \cos x \sin^{n-1} x \Rightarrow \lim_{x \rightarrow 0} \frac{f(x)}{(1-\cos x)^m} = \frac{\ln x \cos x \sin^{n-1} x}{(1-\cos x)^m}$$

$$\Rightarrow \frac{\ln x \cos x (x^2)^{n-1}}{\frac{x^{2m}}{2m}} \Rightarrow \frac{(2m \ln x) (x^{2n-1})}{x^{2m}} \Rightarrow 2m = 2n-1$$

(2)

$$\textcircled{1} \ln x^{m+1} = x^{\omega} \Rightarrow n=2 \Rightarrow m+2 = \omega \Rightarrow m = \omega - 2 \Rightarrow 2m = \omega$$

$$\Rightarrow \omega = 2n-1 \Rightarrow \boxed{n=2} \Rightarrow 2m+n = \omega+2 = 4 \quad \checkmark$$

$$f^{-1}(x) = \left(\frac{x+1}{x-1}\right)^2 \Rightarrow (f^{-1})' = 2 \left(\frac{-1}{(x-1)^2}\right) \left(\frac{x+1}{x-1}\right) \Rightarrow x=2$$

$$\Rightarrow = 2 \times \left(\frac{-1}{1}\right) \left(\frac{3}{1}\right) = -6 \quad \checkmark$$

(2)

$$\frac{f(1)-f(a)}{1-a} = \frac{1-a}{1} = -1 \Rightarrow a = -\frac{1}{2} \Rightarrow f(x) = (x+1)^2 \left(-\frac{1}{2}x+1\right)$$

$$f'(x) = 2x(x+1) \left(-\frac{1}{2}x+1\right) - \frac{1}{2}(x+1)^2$$

$$\Rightarrow f'(1) = 2 \times 2 \times \frac{1}{2} - \frac{1}{2} \times 4 = 2 - 2 = 0 \quad \checkmark$$

(2)

$$f(x) = \sin x \cos x = \frac{1}{2} \sin 2x = \frac{1}{2} (1 - \sin^2 x) = \frac{1}{2} \sin^2 x + \frac{1}{2} \sin x$$

$$g(x) = \sin^2 x - \cos^2 x = \sin^2 x - 1 \Rightarrow \frac{f(\frac{\pi}{4}) - f(\frac{\pi}{2})}{\frac{1}{2} - \frac{1}{2}} = \frac{f(\frac{\pi}{4}) - f(\frac{\pi}{2})}{g(\frac{\pi}{4}) - g(\frac{\pi}{2})}$$

$$\left. \begin{aligned} f(\frac{\pi}{4}) &= \frac{1}{2} + \frac{1}{2} = 1, f(\frac{\pi}{2}) = \frac{1}{2} + \frac{1}{2} = 1 \\ g(\frac{\pi}{4}) &= \frac{1}{2} - 1 = -\frac{1}{2}, g(\frac{\pi}{2}) = 1 - 1 = 0 \end{aligned} \right\} \Rightarrow \frac{1-1}{-\frac{1}{2}-0} = -2 \quad \checkmark$$