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نام و نام خانوادگی ......آرتین جلیلی...... پاسخنامه تشریحی تکلیف شماره ...۲۳... کلاس .دوازدهم تجربی A.......

$$g'(x) = \left(2\tan x(1+\tan^2 x) - \sqrt{2}\sin x\right) f'(\tan^2 x + \sqrt{2}\cos x)$$

$$x=\frac{\pi}{4} \Rightarrow g'\left(\frac{\pi}{4}\right) = 3f'(2) = \sqrt{3} \rightarrow \boxed{f'(2) = \frac{\sqrt{3}}{3}}$$

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$$f(x) = \frac{8+\cos^3 x}{4-\cos^2 x} = \frac{(2+\cos x)(4-2\cos x+\cos^2 x)}{(2-\cos x)(2+\cos x)} = \frac{\cos^2 x - 2\cos x + 4}{2-\cos x} \rightarrow f'(x) = \frac{\sin x\cos x(\cos x - 4)}{(2-\cos x)^2}$$

$$g(x) = \frac{2}{2-\cos x} = 2(2-\cos x)^{-1} \rightarrow g'(x) = -\frac{2\sin x}{(2-\cos x)^2}$$

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$$\rightarrow f'\left(\frac{7\pi}{4}\right) - 2g'\left(\frac{7\pi}{4}\right) = -\frac{3+8\sqrt{2}}{2(4+\sqrt{2})^2} - \frac{8}{(4+\sqrt{2})^2} = \boxed{\frac{-19-8\sqrt{2}}{2(4+\sqrt{2})^2}}$$

$$f(x) = |4x-3|\sqrt{ax} \rightarrow f(x) = \begin{cases} (3-4x)\sqrt{ax}, & x < \frac{3}{4} \\ (4x-3)\sqrt{ax}, & x > \frac{3}{4} \end{cases}$$

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$$f'_-(x) = \frac{(3-4x)a}{2\sqrt{ax}} - 4\sqrt{ax} \rightarrow f'_-\left(\frac{3}{4}\right) = -4\sqrt{\frac{3}{4}a} = -2\sqrt{3a}$$

$$f'_+(x) = \frac{(4x-3)a}{2\sqrt{ax}} + 4\sqrt{ax} \rightarrow f'_+\left(\frac{3}{4}\right) = 4\sqrt{\frac{3}{4}a} = 2\sqrt{3a}$$

$$f'_+\left(\frac{3}{4}\right) - f'_-\left(\frac{3}{4}\right) = 4\sqrt{3a} = 2\sqrt{6} \Rightarrow \sqrt{3a} = \frac{\sqrt{6}}{2} = \frac{\sqrt{3}}{2}$$

$$\rightarrow 3a = \frac{3}{2} \rightarrow \boxed{a = \frac{1}{2}}$$

$$y = -ax+2 \rightarrow f(4) = -4a+2, \quad f'(4) = -a$$

$$f(4) + f'(4) = -4a+2-a = -1 \rightarrow a = \frac{3}{5} \rightarrow f'(4) = -a = \boxed{-\frac{3}{5}}$$

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$$\begin{cases} y = \frac{x+5}{7} \\ y = \frac{ax-1}{3x+1} \end{cases} \rightarrow \frac{x+5}{7} = \frac{ax-1}{3x+1} \rightarrow 3x^2 + (16-7a)x + 12 = 0$$

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چون معادله ریشه مضاعف دارد می توان آن را به صورت مربع کامل نوشت و چون محل تلاقی ناحیه اول است معادله به صورت $3(x-2)^2$ است.

$$3(x-2)^2 = 3x^2 - 12x + 12 = 0 \rightarrow 16 - 7a = -12 \rightarrow 7a = 28 \rightarrow \boxed{a=4}$$

$x=2$ هم ریشه و هم اکسترمم تابع است

$$f'(x)=3x^2+a \Rightarrow f'(2)=12+a=0 \Rightarrow \underline{a=-12}$$

$$f(2)=8+2a-b=8-24-b=0 \Rightarrow \underline{b=-16}$$

$$\Rightarrow b-a=-16+12=\boxed{-4}$$

$$f(x)=\sin^n(x^2) \Rightarrow f'(x)=n\sin^{n-1}(x^2)\times\cos(x^2)\times 2x$$

$\sin(x^2)\sim x^2$, $\cos(x^2)\sim 1$, $1-\cos x\sim \frac{x^2}{2}$

$$f(x)f'(x)=2nx\sin^{2n-1}(x^2)\cos(x^2)\ \underset{\text{هم‌ارزی}}{\overset{x\to 0}{=}}\ 2nx(x^2)^{2n-1}=2nx^{4n-1}$$

$$\Rightarrow \lim_{x\to 0}\frac{f(x)f'(x)}{(1-\cos x)^m}=\frac{2nx^{4n-1}}{\frac{x^{2m}}{2^m}}=2n2^m x^{4n-1-2m}=32\sqrt{2} \Rightarrow 4n-1-2m=0 \Rightarrow 2m=4n-1$$

$$\Rightarrow 2n2^m=32\sqrt{2}=2^{\frac{11}{2}} \xrightarrow{n=2^k} 2^{k+1+m}=2^{\frac{11}{2}} \Rightarrow k+1+m=\frac{11}{2} \Rightarrow m=\frac{4n-1}{2}=\frac{2^{k+2}-1}{2}$$

$$\Rightarrow k=1,\ n=2,\ m=\frac{7}{2}$$

$$\Rightarrow \boxed{2m+n=9}$$

$$y=\frac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow \sqrt{x}y-y=\sqrt{x}+1 \Rightarrow \sqrt{x}y-\sqrt{x}=y+1 \Rightarrow \sqrt{x}(y-1)=y+1$$

$$\sqrt{x}=\frac{y+1}{y-1} \Rightarrow x=\left(\frac{y+1}{y-1}\right)^2 \Rightarrow f^{-1}(x)=\left(\frac{x+1}{x-1}\right)^2$$

مشتق تابع معکوس $\Rightarrow (f^{-1}(x))'=2\left(\frac{x+1}{x-1}\right)\left(\frac{1(x-1)-1(x+1)}{(x-1)^2}\right) \xrightarrow[\text{شیب مماس}]{x=2} m=2(3)(-2)=\boxed{-12}$

آهنگ تغییر متوسط در $[-1,0]$ $=\frac{f(0)-f(-1)}{0-(-1)}=\frac{1-(8(1-a))}{1}=8a-7=-11 \Rightarrow \underline{a=-\frac{1}{2}}$

$$\Rightarrow f(x)=(x^2+1)^3\left(-\frac{1}{2}x+1\right) \Rightarrow f'(x)=3(2x)(x^2+1)^2\left(-\frac{x}{2}+1\right)-\frac{1}{2}(x^2+1)^3$$

در نقطه $x=-2a=1 \Rightarrow f'(1)=\left(3\times 2\times 4\times\frac{1}{2}\right)-4=12-4=\boxed{8}$

$$f(x)=\sin x\cos 2x \Rightarrow \frac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{\sin\frac{\pi}{2}\cos\pi-\sin\frac{\pi}{4}\cos\frac{\pi}{2}}{\frac{\pi}{4}}=\frac{-1}{\frac{\pi}{4}}=\frac{-4}{\pi}$$

$$g(x)=\sin^4x-\cos^4x=(\sin^2x+\cos^2x)(\sin^2x-\cos^2x)=\sin^2x-\cos^2x=-\cos 2x$$

$\Rightarrow \frac{g(\frac{\pi}{2})-g(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{-\cos\pi+\cos\frac{\pi}{2}}{\frac{\pi}{4}}=\frac{1}{\frac{\pi}{4}}=\frac{4}{\pi} \Rightarrow$ آهنگ متوسط تغییر $\frac{f(x)}{g(x)}=\frac{-\frac{4}{\pi}}{\frac{4}{\pi}}=\boxed{-1}$

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$$f'\left(\frac{7\pi}{6}\right)-2g'\left(\frac{7\pi}{6}\right)=(f-2g)'\left(\frac{7\pi}{6}\right)$$

$$\rightarrow (f-2g)'=\left(\frac{8+\cos^3x}{4-\cos^2x}-\frac{4}{2-\cos x}\right)'=\left(\frac{(\cos x+2)(\cos^2x-2\cos x+4)}{(2-\cos x)(2+\cos x)}-\frac{4}{2-\cos x}\right)'=$$

$$\left(\frac{\cos x(\cos x-2)}{-(\cos x-2)}\right)'=-(\cos x)'=\sin x\xrightarrow{x=7\pi/6}\sin\left(\frac{7\pi}{6}\right)=\frac{-1}{2}$$