نام و نام خانوادگی: پوریا پالیزبان .... پاسخنامه تشریحی تکلیف شماره ۷ کلاس دوازدهم ۸

۱

$8\cos x - \tan^2 x = 1 \Rightarrow 8\cos x = 1 + \tan^2 x \Rightarrow 8\cos x = \frac{1}{\cos^2 x} \Rightarrow 8\cos^3 x = 1$

$\Rightarrow \cos x = \frac{1}{2} \Rightarrow x \in [0, 2\pi]$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

۲ جواب دارد.

۲

$5\sin^2 x + 2\cos 3x = -2 \quad \underline{\cos 3x = 4\cos^3 x - 3\cos x} \Rightarrow 5 - 5\cos^2 x + 8\cos^3 x - 6\cos x + 2 = 0$

$5(1-\cos^2 x)$

$\Rightarrow 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0 \Rightarrow$ بر $\cos x + 1$ بخش پذیر است

$$\begin{array}{l|l} 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 & \cos x + 1 \\ \hline 8\cos^3 x + 8\cos^2 x & 8\cos^2 x - 13\cos x + 7 \\ -13\cos^2 x - 6\cos x + 7 & \\ -13\cos^2 x - 13\cos x & \\ 7\cos x + 7 & \\ 7\cos x + 7 & \\ 0 & \end{array}$$

$\Rightarrow (\cos x + 1)(8\cos^2 x - 13\cos x + 7) = 0$

$\Delta = 13^2 - 4\times 8\times 7 = 169 - 224 = -55 \Rightarrow$ ریشه ندارد

$\cos x = -1 \Rightarrow x \in [-\pi, \pi] \Rightarrow x = -\pi, \pi$

۲ جواب

۳

$2\sin x \cos 2x + \sin x = 1 \Rightarrow 2\sin x(1 - 2\sin^2 x) + \sin x = 1$

$\Rightarrow 2\sin x - 4\sin^3 x + \sin x = 1 \Rightarrow 4\sin^3 x - 3\sin x + 1 = 0 \Rightarrow$ این عبارت بر $\sin x + 1$ بخش پذیر است

$$\begin{array}{l|l} 4\sin^3 x - 3\sin x + 1 & \sin x + 1 \\ \hline 4\sin^3 x + 4\sin^2 x & 4\sin^2 x - 4\sin x + 1 = (2\sin x - 1)^2 \\ -4\sin^2 x - 3\sin x + 1 & \\ -4\sin^2 x - 4\sin x & \\ \sin x + 1 & \\ \sin x + 1 & \\ 0 & \end{array}$$

$(\sin x + 1)(2\sin x - 1)^2 = 0$

$\begin{cases} \sin x = -1 \Rightarrow x = \frac{3\pi}{2} \\ \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6} \end{cases} \xrightarrow{+} \frac{3\pi}{2} + \pi = \frac{5\pi}{2}$

۴

$\frac{1}{\sin(\frac{\pi}{2} + 2x)} + \frac{1}{\cos(\frac{\pi}{2} + 4x)} = 0 \Rightarrow \frac{1}{\cos 2x} - \frac{1}{\sin 4x} = 0 \Rightarrow \cos 2x = \sin 4x$

$\Rightarrow \cos 2x = 2\sin 2x \cos 2x$

$\Rightarrow 1 = 2\sin 2x \Rightarrow \sin 2x = \frac{1}{2} \Rightarrow 2x = 2k\pi + \frac{\pi}{6} \Rightarrow x = k\pi + \frac{\pi}{12}$

$2x = 2k\pi + \pi - \frac{\pi}{6} \Rightarrow x = k\pi + \frac{5\pi}{12}$

$\Rightarrow \alpha = \frac{5\pi}{12} - \frac{\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3} = 60^\circ$

ریشه‌ی $\cos 2x$ را بدست نمی‌آوریم چون مخرج را صفر می‌کنند

$\tan 2\alpha = \tan 120^\circ = -\sqrt{3}$

۵

$\cos(x + \frac{\pi}{4}) = \frac{1}{\sqrt{3}} \xrightarrow{\text{توان ۲}} \cos^2(x + \frac{\pi}{4}) = \frac{1 + \cos(2x + \frac{\pi}{2})}{2} = \frac{1}{3} \Rightarrow -\sin 2x = \frac{2}{3} - 1 \Rightarrow \sin 2x = \frac{1}{3}$

$m(\cos x - \sin x) = m(\sqrt{2}\sin(x - \frac{\pi}{4}))$

$\sqrt{2}\sin(\frac{\pi}{4} - x)$

$m\times\sqrt{2}\cos(\frac{\pi}{4} + x) - 3\sqrt{6}\sin 2x = \sqrt{6} \Rightarrow \frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6}$

$\frac{\sqrt{6}}{3}$ ← بر اساس صورت سوال

$\Rightarrow \frac{\sqrt{6}}{3}m = 2\sqrt{6}$

$\Rightarrow m = \frac{2\sqrt{6}}{\frac{\sqrt{6}}{3}} = 6$

$\sin(x+\frac{\pi}{6})\cos(x-\frac{\pi}{3}) = 1 \rightarrow \sin^2(x+\frac{\pi}{6}) = 1 \rightarrow \sin(x+\frac{\pi}{6}) = \pm 1$

$\cos(\frac{\pi}{2}-x)$ مکمل

$\rightarrow x+\frac{\pi}{6} = k\pi + \frac{\pi}{2} \rightarrow x = k\pi + \frac{\pi}{3}$

در بازه $[0, 2\pi]$ ← ۲ جواب دارد.

۴

$\sin x + \sqrt{3}\cos x = \sqrt{2} \xrightarrow{\times \frac{1}{2}} \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2} \rightarrow \sin(x+\frac{\pi}{3}) = \frac{\sqrt{2}}{2}$

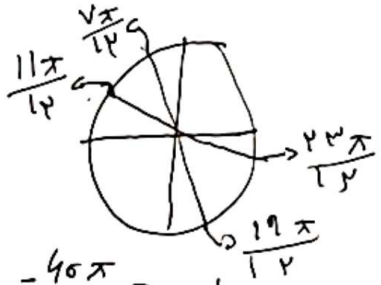
$x+\frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \rightarrow x = 2k\pi - \frac{\pi}{12}$

$x+\frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \rightarrow x = 2k\pi + \frac{5\pi}{12}$ $\xrightarrow{[0, 2\pi]}$ $x = \frac{5\pi}{12}, -\frac{\pi}{12}, \frac{23\pi}{12}$

مجموع $\frac{23\pi}{12}$

۷

$4\sin x \underbrace{\sin(\frac{3\pi}{2}-x)}_{-\cos x} = 1 \rightarrow 4\sin x\cos x = -1 \rightarrow 2\sin 2x = -1 \rightarrow \sin 2x = -\frac{1}{2}$



$2x = 2k\pi - \frac{\pi}{6} \rightarrow x = k\pi - \frac{\pi}{12}$

$2x = 2k\pi - \frac{5\pi}{6} \rightarrow x = k\pi - \frac{5\pi}{12}$

مجموع جواب‌ها $= \frac{7\pi}{12} + \frac{11\pi}{12} + \frac{19\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = 5\pi$

۱۰

$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha) = \frac{1}{8} \rightarrow \cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{64}$

$2\cos^2\alpha \quad 2\cos^2 2\alpha \quad 2\cos^2 4\alpha$

$\rightarrow \cos\alpha\cos 2\alpha\cos 4\alpha = \pm\frac{1}{8} \xrightarrow{\times \frac{\sin\alpha}{\sin\alpha}} \frac{\frac{1}{2}\sin 2\alpha \cdot \frac{1}{2}\sin 4\alpha \cdot \frac{1}{2}\sin 8\alpha}{\sin\alpha} = \pm\frac{1}{8}$

$\rightarrow \frac{1}{8}\frac{\sin 8\alpha}{\sin\alpha} = \pm\frac{1}{8} \rightarrow \sin 8\alpha = \pm\sin\alpha$

$\sin 8\alpha = +\sin\alpha \rightarrow \alpha = \frac{2k\pi}{7}$ و $\frac{2k\pi+\pi}{9}$

$\sin 8\alpha = -\sin\alpha \rightarrow \alpha = \frac{2k\pi}{9}$ و $\frac{2k\pi+\pi}{7}$

جواب‌ها: $\frac{\pi}{9}$ و $\frac{\pi}{7}$ و π و $\frac{3\pi}{7}$ و $\frac{5\pi}{9}$ و $\frac{7\pi}{9}$

$\sin$ چون $\frac{5\pi}{9}$ و $\frac{7\pi}{9}$ → صفر می‌شود

۹

$\tan^2 x \tan x = 1 \rightarrow \tan 2x = \frac{1}{\tan x} \rightarrow \tan 2x = \tan(\frac{\pi}{2}-x)$

$2x = k\pi + \frac{\pi}{2} - x \Rightarrow 3x = k\pi + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{3} + \frac{\pi}{6}$

$k=0 \rightarrow x = \frac{\pi}{6}$ $\quad k=1 \rightarrow x = \frac{3\pi}{6}$

$k=2 \rightarrow x = \frac{5\pi}{6}$ $\quad k=3 \rightarrow x = \frac{7\pi}{6}$

$k=4 \rightarrow x = \frac{9\pi}{6}$ $\quad k=5 \rightarrow x = \frac{11\pi}{6}$

مجموع جواب‌ها $= \frac{\pi}{6} + \frac{3\pi}{6} + \frac{5\pi}{6} + \frac{7\pi}{6} + \frac{9\pi}{6} + \frac{11\pi}{6} = \frac{36\pi}{6} = 6\pi$

۱۰