نام و نام خانوادگی: حسام رفیع زاده

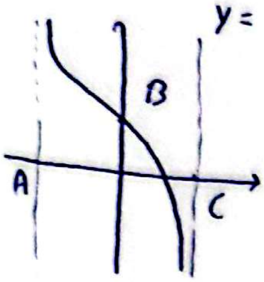
« به نام خدا »

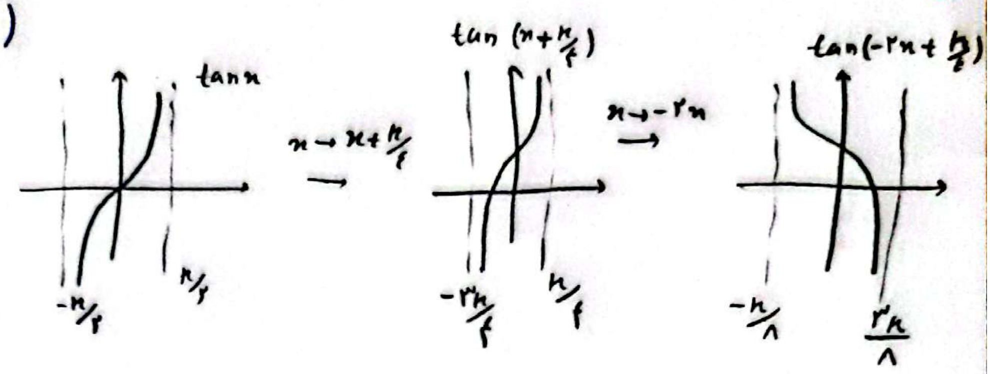
مقطع: دوازدهم پسر A

شماره: ۱۵

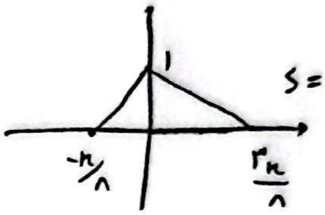
-۱

$y = \tan(-2x + \frac{\pi}{4})$





$\longrightarrow A = -\frac{\pi}{8}$, $\boxed{C = \frac{3\pi}{8}}$, $x = 0 \to \tan(\frac{\pi}{4}) = 1 \to \boxed{B = 1}$



$S = \frac{1 \times \frac{\pi}{2}}{2} = \boxed{\frac{\pi}{4}}$

-۲

$x = \frac{2}{3} \xrightarrow[\text{در معادله}]{\text{صدق}} \frac{4}{9} - \frac{2}{3}\sin\alpha - \frac{1}{4}\cos^2\alpha = 0 \xrightarrow{\times 36} 16 - 24\sin\alpha - 9\cos^2\alpha = 0 \to 16 - 24\sin\alpha - 9(1 - \sin^2\alpha) = 0$

$\to 9\sin^2\alpha - 24\sin\alpha + 7 = 0 \xrightarrow[\text{روسی}]{\text{روش}} (\sin\alpha - 21)(\sin\alpha - 3) = 0 \to \sin\alpha = \frac{1}{3}$ ①

$\hookrightarrow \frac{21}{9}$ غ ق ق $\qquad \hookrightarrow \frac{3}{9}$ ق ق

$\sin^2\alpha + \cos^2\alpha = 1 \xrightarrow{①} \cos^2\alpha = \frac{8}{9}$ ②

$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} \overset{②}{\Longrightarrow} \boxed{1 + \tan^2\alpha = \frac{9}{8}}$

-۳

$\log \frac{\sin\alpha}{-\cos\alpha} = \log 3 \to \frac{\sin\alpha}{-\cos\alpha} = 3 \to \sin\alpha = -3\cos\alpha$ ①

$\sin^2\alpha + \cos^2\alpha = 1 \xrightarrow{①} 9\cos^2\alpha + \cos^2\alpha = 1 \longrightarrow \cos\alpha = \frac{\sqrt{10}}{10}$, $\sin\alpha = \frac{-3\sqrt{10}}{10}$

$\sin\alpha - \cos\alpha = \frac{-3\sqrt{10}}{10} - \frac{\sqrt{10}}{10} = \boxed{-0.4\sqrt{10}}$

۴-

$$\sin^4\theta + \cos^2\theta = \frac{4}{5} \quad , \quad \cos^4\theta + \sin^2\theta = ?$$

$$\sin^4\alpha + \cos^2\alpha = \frac{4}{5} \rightarrow \sin^4\alpha - \sin^2\alpha + 1 = \frac{4}{5} \rightarrow \sin^4\alpha - \sin^2\alpha = \frac{-1}{5} \; (1) \quad , \quad \sin^2\alpha = \sin^4\alpha + \frac{1}{5} \; (2)$$

$$\cos^4\alpha + \sin^2\alpha = (1-\sin^2\alpha)^2 + \sin^2\alpha \xrightarrow{(2)} (1-\sin^2\alpha)^2 + \sin^4\alpha + \frac{1}{5} = 2\sin^4\alpha - 2\sin^2\alpha + \frac{6}{5}$$

$$\xrightarrow{(1)\times 2} \frac{-2}{5} + \frac{6}{5} = \boxed{\frac{4}{5}}$$

۵-

$$\frac{3\sin\alpha + \sin\alpha \cdot \cos\alpha}{2 - 2\cos^2\alpha} < 0 \rightarrow \frac{\sin\alpha\,(3+\cos\alpha)}{2(1-\cos^2\alpha)} < 0 \rightarrow \frac{\sin\alpha\,(3+\cos\alpha)}{2\sin^2\alpha} < 0$$

$$\rightarrow \frac{3+\cos\alpha}{2\sin\alpha} < 0 \xrightarrow{-1 \leq \cos\alpha \leq 1} \frac{\overbrace{3+\cos\alpha}^{\text{همواره مثبت}}}{\underbrace{2\sin\alpha}_{\text{باید منفی باشد}}} < 0 \rightarrow \sin\alpha < 0$$

 (1)

$$\sin\alpha \cdot \tan\alpha - \frac{1}{\cos\alpha} > 0 \rightarrow \frac{\sin^2\alpha - 1}{\cos\alpha} > 0 \xrightarrow{\times -1} \frac{1-\sin^2\alpha}{\cos\alpha} < 0 \rightarrow \frac{\cos^2\alpha}{\cos\alpha} < 0 \rightarrow \cos\alpha < 0$$

 (2)

$\xrightarrow{(1)\cap(2)}$

α ناحیه‌ی سوم

۶-

$$\frac{2}{\sin\alpha} + \frac{3}{\cos\alpha} = 0 \quad , \quad \tan\alpha + \cot\alpha = ?$$

$$\frac{2\cos\alpha + 3\sin\alpha}{\sin\alpha \cdot \cos\alpha} = 0 \xrightarrow[\cos\alpha \neq 0]{\sin\alpha \neq 0} 2\cos\alpha + 3\sin\alpha = 0 \rightarrow 2\cos\alpha = -3\sin\alpha \rightarrow \cos\alpha = \frac{-3}{2}\sin\alpha \; (1)$$

$$\sin^2\alpha + \cos^2\alpha = 1 \rightarrow \sin^2\alpha + \frac{9}{4}\sin^2\alpha = 1 \rightarrow \frac{13}{4}\sin^2\alpha = 1 \rightarrow \sin^2\alpha = \frac{4}{13} \rightarrow \sin\alpha = \frac{2}{\sqrt{13}} \; , \; \cos\alpha = \frac{-3}{\sqrt{13}}$$

$$\tan\alpha + \cot\alpha = \frac{1}{\sin\alpha \cdot \cos\alpha} = \frac{1}{\frac{2}{\sqrt{13}} \times \frac{-3}{\sqrt{13}}} = \boxed{\frac{-13}{6}}$$

-۷

$$\frac{\sqrt{1+\sin 50^\circ}}{\sin 50^\circ+\sin 10^\circ}=? \quad , \quad \sin(60-10)=\sin 50^\circ \to \frac{\sqrt{3}}{2}\cos 10-\frac{1}{2}\sin 10=\sin 50^\circ \quad ①$$

$$\frac{\sqrt{1+2\sin 25^\circ\cdot\cos 25^\circ}}{\sin 50^\circ+\sin 10^\circ}=\frac{\sin 25^\circ+\cos 25^\circ}{\sin 50^\circ+\sin 10^\circ}\overset{①}{=}\frac{\sin 25^\circ+\cos 25^\circ}{\frac{\sqrt{3}}{2}\cos 10^\circ-\frac{1}{2}\sin 10^\circ+\frac{1}{2}\sin 10^\circ}=\frac{\sin 25^\circ+\cos 25^\circ}{\frac{1}{2}\sin 10^\circ+\frac{\sqrt{3}}{2}\cos 10^\circ}$$

$$\to \frac{\sin 25^\circ+\cos 25^\circ}{\sin 30^\circ\cdot\sin 10^\circ+\cos 30^\circ\cdot\cos 10^\circ}=\frac{\sin 25^\circ+\cos 25^\circ}{\cos(30^\circ-10^\circ)}=\frac{\sqrt{2}\sin(25+45)}{\cos 20^\circ}=\frac{\sqrt{2}\sin 70^\circ}{\cos(90-70^\circ)}$$

$$=\frac{\sqrt{2}\sin 70^\circ}{\sin 70^\circ}=\boxed{\sqrt{2}}$$

-۸

$$(1-2\sin^2 5^\circ)(2\cos^2 10^\circ-1)\left(\frac{1-\tan^2 20}{1+\tan^2 20^\circ}\right)$$

پرانتز اول: $1-2\sin^2 5^\circ=\cos 10^\circ$

پرانتز دوم: $2\cos^2 10^\circ-1=\cos 20^\circ$

پرانتز سوم: $\frac{1-\tan^2 20}{1+\tan^2 20}=\frac{1-\tan^2 20}{\frac{1}{\cos^2 20}}=\cos^2 20^\circ\times\frac{\cos^2 20^\circ-\sin^2 20^\circ}{\cos^2 20^\circ}=\cos 40^\circ$

$$\xrightarrow{\text{ضرب در هم}} \cos 10^\circ\times\cos 20^\circ\times\cos 40^\circ \xrightarrow{\times\sin 10^\circ} \underbrace{\sin 10^\circ\times\cos 10^\circ}_{\frac{1}{2}\sin 20^\circ}\times\cos 20^\circ\times\cos 40^\circ=\frac{1}{4}\sin 40^\circ\times\cos 40^\circ$$

$$\frac{1}{2}\sin 20^\circ\times\cos 20^\circ = \frac{1}{4}\sin 40^\circ$$

$$=\frac{1}{8}\sin 80^\circ \xrightarrow{\div\sin 10^\circ} \frac{1}{8}\,\frac{\sin 80^\circ}{\sin 10^\circ}$$

$$=\frac{1}{8}\,\frac{\sin(90-10)}{\sin 10}=\boxed{\frac{1}{8}\cot 10^\circ}$$

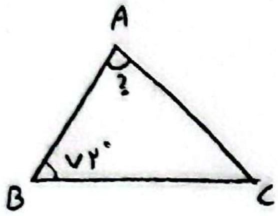
$\sin\alpha + 3\cos\alpha = 2 \,,\; 5\cos 2\alpha - 12\cos\alpha = ?$

$\sin\alpha = 2 - 3\cos\alpha$ (1)

$5(\cos^2\alpha - \sin^2\alpha) - 12\cos\alpha = 5(2\cos^2\alpha - 1) - 12\cos\alpha = 10\cos^2\alpha - 12\cos\alpha - 5 = ?$

$\sin^2\alpha + \cos^2\alpha = 1 \xrightarrow{(1)} (2 - 3\cos\alpha)^2 + \cos^2\alpha = 1 \rightarrow 10\cos^2\alpha - 12\cos\alpha = -3$ (2)

$10\cos^2\alpha - 12\cos\alpha - 5 \xrightarrow{(2)} -3 - 5 = \boxed{-8}$



$\sin\hat{B}\cdot\sin\hat{C} = \cos^2\frac{\hat{A}}{2} \,,\; \cos A = 2\cos^2\frac{\hat{A}}{2} - 1 \rightarrow \cos^2\frac{\hat{A}}{2} = \frac{\cos A + 1}{2}$ ۱۰-

$\sin\hat{B}\cdot\sin\hat{C} = \frac{\cos A + 1}{2} \rightarrow 2\sin\hat{B}\cdot\sin\hat{C} = \cos A + 1$

$\rightarrow 2\sin\hat{B}\cdot\sin\hat{C} = \cos(180 - (B+C)) + 1 \rightarrow 1 - 2\sin\hat{B}\cdot\sin\hat{C} = \cos(\hat{B}+\hat{C})$

$1 - 2\sin\hat{B}\cdot\sin\hat{C} = \cos\hat{B}\cdot\cos\hat{C} - \sin\hat{B}\cdot\sin\hat{C} \rightarrow 1 - 2\sin\hat{B}\cdot\sin\hat{C} + \sin\hat{B}\cdot\sin\hat{C} = \cos\hat{B}\cdot\cos\hat{C}$

$\rightarrow 1 - \sin\hat{B}\cdot\sin\hat{C} = \cos\hat{B}\cdot\cos\hat{C} \rightarrow \cos\hat{B}\cdot\cos\hat{C} + \sin\hat{B}\cdot\sin\hat{C} = 1$

$\rightarrow \cos(\hat{B} - \hat{C}) = 1 \rightarrow \hat{B} - \hat{C} = 0 \rightarrow \hat{B} = \hat{C} \rightarrow \hat{B} = 72 ,\ \hat{C} = 72 \rightarrow \boxed{\hat{A} = 36^\circ}$

یا $\hat{B} - \hat{C} = 180$ غق

$\rightarrow \hat{C} - \hat{B} = 180 \rightarrow \hat{C} > 180$ غق
