

Diagram of a triangle with sides r and r , and angle α . The third side is labeled $2r \sin \frac{\alpha}{2}$.

$$\frac{r}{\sin \alpha} + \frac{r}{\cos \alpha} = 1 \Rightarrow r \cos \alpha = -r \sin \alpha$$

$$\Rightarrow \cos \alpha = -\frac{r}{r}$$

$$\Rightarrow \cos \alpha = -\frac{r}{r}$$

$$2r \sin \alpha + \cos \alpha = -\frac{r}{r} + (-\frac{r}{r}) = -\frac{2r}{r}$$

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(P) ✓

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$$\sqrt{1 + \sin^2 \alpha} = \sqrt{1 + \cos^2 \alpha} = \sqrt{2} \cos \alpha$$

$$\sin \alpha + \sin \alpha = \sin(\alpha + \alpha) + \sin(\alpha - \alpha)$$

$$\frac{\sqrt{2} \cos \alpha}{2 \sin \alpha \cos \alpha} = \sqrt{2}$$

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$$(1 - r \sin^2 \alpha) (r \cos^2 \alpha - 1) \left(\frac{1 - \sin^2 \alpha}{1 + \sin^2 \alpha} \right) = \cos^2 \alpha \times \cos^2 \alpha \times \cos^2 \alpha =$$

$$\times \frac{\sin^2 \alpha}{\sin^2 \alpha}$$

$$\frac{1}{r} \frac{\sin^2 \alpha \cos^2 \alpha \times \cos^2 \alpha}{\sin^2 \alpha} = \frac{1}{r} \frac{\sin^2 \alpha \cos^2 \alpha}{\sin^2 \alpha} \rightarrow \frac{1}{r} \frac{\sin^2 \alpha}{\sin^2 \alpha}$$

$$= \frac{1}{r} \frac{\cos^2 \alpha}{\sin^2 \alpha} = \frac{1}{r} \cot^2 \alpha = 1 \cot^2 \alpha \quad (P) \checkmark$$

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$$\sin \alpha + r \cos \alpha = r$$

$$\sin \alpha + r - r \cos \alpha \rightarrow (\sin \alpha)^2 = (r - r \cos \alpha)^2$$

$$\sin^2 \alpha = r^2 + r^2 \cos^2 \alpha - 2r^2 \cos \alpha$$

$$1 - \cos^2 \alpha = r^2 + r^2 \cos^2 \alpha - 2r^2 \cos \alpha$$

$$1 \cdot \cos^2 \alpha - r^2 \cos^2 \alpha + r^2 = -1$$

$$1 \cdot \frac{1 + \cos^2 \alpha}{r} - r \cos^2 \alpha + r^2 =$$

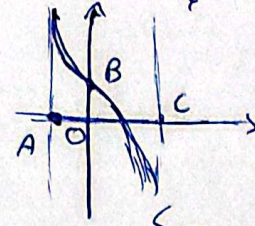
$$\frac{1 + \cos^2 \alpha}{r} = \frac{1 + \cos^2 \alpha}{r}$$

$$r \sin \alpha \sin \alpha = 1 + \cos^2 \alpha \quad A+B+C=\pi$$

$$r \sin \alpha \cos \alpha = 1 + \cos^2 (\pi - \alpha + \alpha) \dots$$

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جانب اول: $A - 2r + \frac{\pi}{r} = \frac{\pi}{r} \Rightarrow -2r = \frac{\pi}{r} \Rightarrow r = -\frac{\pi}{2} \Rightarrow A(-\frac{\pi}{2}, 0)$ جانب دوم: $C - 2r + \frac{\pi}{r} = -\frac{\pi}{r} \Rightarrow -2r = -\frac{2\pi}{r} \Rightarrow r = \frac{\pi}{2} \Rightarrow C(\frac{\pi}{2}, 0)$ نقطه B: $r = 0 \Rightarrow \tan \frac{\pi}{2} = 1 \Rightarrow B(0, 1)$ $S_{ABC} = \frac{AC \times OB}{2} = \frac{\pi \times 1}{2} = \frac{\pi}{2}$ (پ) ✓ 	۱
$r - (\sin u) r - \frac{1}{r} \cos u = 0 \Rightarrow \frac{r}{a} - \frac{r}{r} \sin u - \frac{1}{r} + \frac{1}{r} \sin u = 0 \Rightarrow$ $(1 - \sin u) = 0$ (پ) ✓ $\frac{1}{r} \sin u - \frac{r}{r} \sin u + \frac{1}{r} = 0 \Rightarrow a \sin u - r \sin u + 1 = 0 \Rightarrow \sin u = \frac{r - 1}{a}$ $\Rightarrow \sin u = \frac{1}{r} \Rightarrow \sin u = \frac{1}{a} \Rightarrow \cos u = \frac{1}{a} \Rightarrow \frac{1}{\cos u} = \frac{a}{1} = 1 + \tan u$	۲
$\log(\sin u) - \log(-\cos u) = \log r \Rightarrow \log -\tan u = \log r \Rightarrow -\tan u = r$ $\sin u > 0, -\cos u > 0 \Rightarrow \cos u < 0 \Rightarrow \frac{\sin u}{-\cos u} = r \Rightarrow \sin u = -r \cos u$ $\Rightarrow \sin^2 u + \cos^2 u = 1 \Rightarrow a \cos^2 u + r^2 \cos^2 u = 1 \Rightarrow \cos^2 u = \frac{1}{1 + r^2} \Rightarrow \cos u = \frac{1}{1 + r^2}$ $\Rightarrow \sin u = \frac{r}{1 + r^2} \Rightarrow \sin u - \cos u = \frac{r}{1 + r^2} - \frac{1}{1 + r^2} = \frac{r - 1}{1 + r^2}$ (پ) ✓	۳
$\sin^2 \theta + \cos^2 \theta = \frac{r}{a} \Rightarrow \sin^2 \theta - \sin^2 \theta + 1 = \frac{r}{a} \Rightarrow \sin^2 \theta (\sin^2 \theta - 1) = -\frac{1}{a}$ $-\sin^2 \theta \cos^2 \theta = -\frac{1}{a}$ (I) / $\cos^2 \theta + \sin^2 \theta = \cos^2 \theta (\cos^2 \theta - 1) + 1 =$ $-\sin^2 \theta \cos^2 \theta + 1 \xrightarrow{I} -\frac{1}{a} + 1 = \frac{r}{a}$ (پ) ✓	۴
$\frac{r \sin u + \sin u \cos u}{r - r \cos^2 u} < 0 \Rightarrow \frac{\sin u (r + \cos u)}{r (1 - \cos^2 u)} < 0 \Rightarrow \frac{r + \cos u}{r \sin u} < 0$ $\frac{r + \cos u}{r \sin u} < 0 \Rightarrow \frac{r + \cos u}{r \sin u} < 0$ $\frac{r + \cos u}{r \sin u} < 0 \Rightarrow \frac{r + \cos u}{r \sin u} < 0$ (پ) ✓	۵