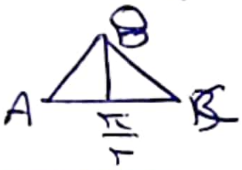
کامیار بشیری دوازدهم جبر B

(1

$y=\tan(-2x+\frac{\pi}{4}) \Rightarrow \frac{\pi}{|a|}=$ دوره تناوب

$\Rightarrow \frac{\pi}{|-2|}=\frac{\pi}{2} \Rightarrow AC=\frac{\pi}{2}$

$B \longrightarrow \tan(-2(0)+\frac{\pi}{4})=\tan(\frac{\pi}{4})=1$

$\Rightarrow$ 

$\frac{1\times\frac{\pi}{2}}{2}=\frac{\pi}{4}=S$ مثلث

(2

$x^2-\sin\alpha\, x-\frac{1}{4}+\frac{\sin^2\alpha}{4}=0$

$\underset{\text{ریشه}}{\frac{2}{3}} \longrightarrow \frac{4}{9}-\frac{2\sin\alpha}{3}-\frac{1}{4}+\frac{\sin^2\alpha}{4}=0$

$\frac{4-6\sin\alpha}{9}+\frac{-1+\sin^2\alpha}{4}=0$

$36\times\frac{16-24\sin\alpha+9+9\sin^2\alpha}{36}=0\times 36$

$9\sin^2\alpha-24\sin\alpha+7=0$

$\Rightarrow \sin^2\alpha-24\sin\alpha+63=0$

$\Rightarrow (\sin\alpha-21)(\sin\alpha-3)=0$

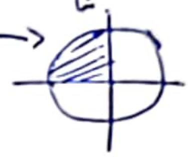
$\sin\alpha=\frac{21}{9}$ غ ق ق $\qquad \sin\alpha=\frac{3}{9}=\frac{1}{3}$ قابل قبول

$\Rightarrow 1-\sin^2\alpha=\cos^2\alpha \Rightarrow \frac{9}{9}-\frac{1}{9}=\frac{8}{9} \Rightarrow \cos^2\alpha$

$\Rightarrow 1+\tan^2\alpha=\frac{1}{\cos^2\alpha}=\frac{9}{8} \Rightarrow \frac{9}{8}$

$\log(\sin\alpha) - \log(-\cos\alpha) = \log 3$

چون log‌ها تعریف شده است
پس حتماً ربع دوم بوده است



$\Rightarrow \log\left(-\dfrac{\sin\alpha}{\cos\alpha}\right) = \log 3 \Rightarrow \tan\alpha = -3$

(۳

$\Rightarrow 1+\tan^2\alpha = \dfrac{1}{\cos^2\alpha} \Rightarrow 1+9 = 10 \Rightarrow \cos^2\alpha = \dfrac{1}{10} \quad \cos\alpha = -\dfrac{1}{\sqrt{10}}$

$\Rightarrow \dfrac{10}{10} - \dfrac{1}{10} = \dfrac{9}{10} = \sin^2\alpha \Rightarrow \dfrac{3}{\sqrt{10}} = \sin\alpha$

$\Rightarrow \sin\alpha - \cos\alpha = \dfrac{3}{\sqrt{10}} + \dfrac{1}{\sqrt{10}} = \dfrac{4}{\sqrt{10}}$

(

$\sin^4\alpha + \cos^2\alpha = \dfrac{4}{5}$

$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow (\sin^2\alpha)^2 + \cos^2\alpha = \dfrac{4}{5} \qquad \sin^2\alpha = x$

$\Rightarrow (x)^2 + (1-x) = \dfrac{4}{5} \qquad x^2 - x + \dfrac{1}{5} = 0$

(۴

$5x^2 - 5x + 1 = 0 \qquad x = \dfrac{5 \pm \sqrt{5}}{10}$

$\Delta = b^2 - 4ac = 25 - 20 = 5$

$\Rightarrow \sin^2\alpha = \dfrac{5 \pm \sqrt{5}}{10} \Rightarrow \cos^2\alpha = 1 - \dfrac{5 \pm \sqrt{5}}{10}$

$\Rightarrow \cos^4\alpha + \sin^2\alpha \Rightarrow = \left(\dfrac{5 \mp \sqrt{5}}{10}\right)^2 + \dfrac{5 \pm \sqrt{5}}{10}$

$\dfrac{30 - 10\sqrt{5}}{100} + \dfrac{50 + 10\sqrt{5}}{100} = \dfrac{80}{100} = \dfrac{4}{5}$

در حالت منفی و مثبت جواب ۴/۵ را هم که امتحان کنی همین
بدست خواهد آمد.

$$\frac{3\sin\alpha + \sin\alpha\cos\alpha}{2(\cancel{1-\cos^2\alpha})_{\sin^2\alpha}} < 0 \Rightarrow 3\sin\alpha + \sin\alpha\overset{+}{\cos\alpha} < 0$$

$$3\cancel{\sin\alpha} < -\cancel{\sin\alpha}\cos\alpha$$

$$3 < -\cos\alpha$$

(۵

$$(-3 > \cos\alpha)$$

$$\frac{(\sin^2\alpha - 1)}{\cos\alpha} > 0$$

$$\sin\alpha(3+\cos\alpha) < 0$$

$$(\cos\alpha > 0)$$

$$\xrightarrow[+]{(\cos\alpha > 0)} \Rightarrow \sin\alpha < 0$$

$$\Rightarrow$$ 

ربع ۴

$$\tan\alpha + \cot\alpha = \underline{\quad ? \quad}$$

(۶

$$\Rightarrow \frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha} = \frac{\sin^2\alpha + \cos^2\alpha}{\cos\alpha\sin\alpha} = \frac{1}{\cos\alpha\sin\alpha}$$

$$\Rightarrow \frac{2\cos\alpha + 3\sin\alpha}{\sin\alpha\cos\alpha} = 0 \qquad 2\cos\alpha + 3\sin\alpha = 0$$

$$2\cos\alpha = -3\sin\alpha$$

$$\tan\alpha + \cot\alpha =$$

$$-\frac{2}{3} - \frac{3}{2} = \frac{-4-9}{6} = \frac{-13}{6} \Leftarrow \left\} \begin{array}{l} \Rightarrow \dfrac{\cos\alpha}{\sin\alpha} = \dfrac{-3}{2} \\ \Rightarrow \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{2}{3} \end{array}\right.$$

(۷

$$\frac{\sqrt{1+2\sin 25\cos 25}}{\cos 40+\sin 10}=\frac{\sqrt{\sin^2 25+\cos^2 25+2\sin 25\cos 25}}{\cos 40+\sin 10}$$

$$\frac{\sin 25+\cos 25}{\cos(60-20)+\sin(30-20)}=\frac{\sin 25+\cos 25}{\frac{1}{2}\cos 20+\frac{\sqrt{3}}{2}\sin 20+\frac{1}{2}\cos 20-\frac{\sqrt{3}}{2}\sin 20}=\frac{\sin 25+\cos 25}{\cos 20}$$

$$\frac{\sqrt{2}\sin(\cancel{45+25})^{70}}{\cancel{\cos 20}}=\sqrt{2}$$

(۸

$$\cancel{2\cos^2 10-1-4\sin^2 5\cos^2 10-2\sin^2 5}$$

$$(1-2\sin^2 5)(2\cos^2 10-1)\left(\frac{1-\tan^2 20}{1+\tan^2 20}\right)=(\cos 10)(\cos 20)(\cos 40)$$

$$\frac{\frac{1}{2}\sin 20\cos 20\cos 40}{\sin 10}=\frac{\frac{1}{4}\left(\frac{1}{2}\sin 40\right)\cos 40}{\sin 10}=\frac{\frac{1}{4}\times\frac{1}{2}(\sin 80)}{\sin 10}$$

$$\Rightarrow \frac{1}{8}\frac{\cos 10}{\sin 10}=\frac{1}{8}\cot 10 \checkmark$$

(۹

$$3\cos\alpha=2-\sin\alpha \;\Rightarrow\; 3\cos\alpha-2=\sin\alpha$$

$$\Rightarrow (3\cos\alpha-2)^2+\cos^2\alpha=1 \quad 9\cos^2\alpha+4-12\cos\alpha+\cos^2\alpha=1$$

$$10\cos^2\alpha-12\cos\alpha+3=0 \;\Rightarrow\; 10\cos^2\alpha-12\cos\alpha=-3$$

$$\Rightarrow 5\cos 2\alpha=5(2\cos^2\alpha-1)=10\cos^2\alpha-5$$

$$\Rightarrow \underbrace{\cancel{10\cos^2\alpha-12\cos\alpha}}_{-3}-5=-3-5=-8$$

با تفکر در فرمول های مثلثاتی تنها فرمولی که ضرب ۲ زاویه متفاوت درهم در آنها دیده می شود $\cos(\alpha\pm\beta)$ و $\sin(\alpha\pm\beta)$ است پس
و فرمولی لازم که sin های لازم باشد cos ها باهم متفاوت در $\cos(\alpha\pm\beta)$ است

$$\sin\hat{B}\sin\hat{C} \Rightarrow \cos B\cos C + \sin C\sin B - \underbrace{\cos B\cos C + \sin C\sin B}$$

چون باید $\cos B\cos C$ را از طرفین حذف کنیم

$$\Rightarrow \sin\hat{B}\sin\hat{C} = \frac{1}{۲}\left(\cos(B-C) - \cos(B+C)\right) = \cos^{۲}\frac{A}{۲}$$

$$\cos^{۲}\frac{A}{۲} = \frac{1+\cos A}{۲} \Rightarrow \frac{1}{۲}\left(\cos(B-C) - \cos(B+C)\right) = \frac{1+\cos A}{۲}$$

$$\Rightarrow \cos(B-C) - \cos(\underbrace{\cancel{B+C}}_{۱۸۰-A}) = 1+\cos A$$

۱۰

$$\Rightarrow A+B+C = ۱۸۰ \Rightarrow B+C = ۱۸۰-A$$

$$\cos(B-C) - \underbrace{\cancel{\cos(\pi - A)}}_{-\cos(A)} = \cos(B-C) + \cancel{\cos A} = 1 + \cancel{\cos A}$$

$$\cos(B-C) = 1 \qquad \cos(0^\circ) = 1$$

$$B-C = 0 \qquad \cos(۳۶۰^\circ) = 1$$

$B-C = ۳۶۰$ غ ق ق

$$\Rightarrow B = C \qquad B = ۷۲ \Rightarrow C = ۷۲$$

$$\Rightarrow ۷۲ + ۷۲ = ۱۴۴ \Rightarrow ۱۸۰ - ۱۴۴ = \underline{۳۶ = \hat{A}}$$

جواب