(۷)

$$\frac{\sqrt{1+\sin 50^\circ}}{\sin 50^\circ + \sin 10^\circ} = \frac{\sqrt{(\sin 25^\circ + \cos 25^\circ)^2}}{\sin 30^\circ\cos 20^\circ + \sin 20^\circ\cos 30^\circ + \sin 30^\circ\cos 20^\circ - \sin 20^\circ\cos 30^\circ} = \frac{\sin 25^\circ + \cos 25^\circ}{2\sin 30^\circ \cos 20^\circ}$$

$\sin(30+20) \quad \sin(30-20)$, $\quad \sin 30^\circ = \frac{1}{2}$

$$= \frac{\sqrt{2}\sin(25^\circ+45^\circ)}{\cos 20^\circ} = \frac{\sqrt{2}\sin 70^\circ}{\cos 20^\circ} = \sqrt{2}$$

(۲) ✓

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(۸)

$$(1-2\sin^2 5^\circ)(2\cos^2 10^\circ - 1)\left(\frac{1-\tan^2 20^\circ}{1+\tan^2 20^\circ}\right) = \cos 10^\circ \cos 20^\circ \cos 40^\circ \times \frac{\sin 10^\circ}{\sin 10^\circ} = \frac{\sin 10^\circ \cos 10^\circ \cos 20^\circ \cos 40^\circ}{\sin 10^\circ}$$

$1-2\sin^2 5^\circ = \cos 10^\circ$, $\quad 2\cos^2 10^\circ - 1 = \cos 20^\circ$, $\quad \frac{1-\tan^2 20^\circ}{1+\tan^2 20^\circ} = \cos 40^\circ$, $\quad \sin 10^\circ\cos 10^\circ = \frac{1}{2}\sin 20^\circ$

$$= \frac{1}{2}\,\frac{\sin 20^\circ \cos 20^\circ \cos 40^\circ}{\sin 10^\circ} \longrightarrow \frac{1}{2}\times\frac{1}{2}\times\frac{\sin 40^\circ \cos 40^\circ}{\sin 10^\circ} = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{\sin 80^\circ}{\sin 10^\circ} = \frac{1}{8}\cot 10^\circ$$

(۲) ✓

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(۹)

$$\sin x + 3\cos x = 2 \xrightarrow{①} \sin x = 2 - 3\cos x \xrightarrow{\text{بتوان ۲}} \sin^2 x = 4 - 12\cos x + 9\cos^2 x$$

$\sin^2 x = 1-\cos^2 x$

$$= 10\cos^2 x + 3 - 12\cos x = 0$$

(۲) ✓

$$②\ \ 5\cos 2x - 12\cos x = 5(2\cos^2 x - 1) - 12\cos x = 10\cos^2 x - 5 - 12\cos x$$

$$10\cos^2 x + 3 - 12\cos x = 0 \xrightarrow{-8} 10\cos^2 x - 5 - 12\cos x = \boxed{-8}$$

عبارت ① و ② که یکی از دو جواب است، ۸ تا اختلاف دارند

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(۱۰)

$$\sin\hat{B}\sin\hat{C} = \cos^2\frac{\hat{A}}{2}, \quad \hat{B} = 72^\circ$$

$$\hookrightarrow \sin\hat{B}\sin\hat{C} = \frac{1+\cos A}{2}$$

$$2\sin B\sin C = 1 + \cos\hat{A} \xrightarrow{A+B+C=\pi}$$

$$2\sin B\sin C = 1 + \cos(\pi - (B+C))$$

$$2\sin B\sin C = 1 - \cos(B+C)$$

$$\rightarrow \sin B\sin C + \cos B\cos C = 1$$

$$\cos(B-C) = 1 \rightarrow B = C$$

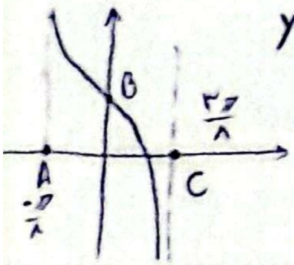
$$180 - (72+72) = 36$$

۵

آرا آزادنیا — تکلیف ۱۵ — دوازدهم میان A

۱۸

①



$y=\tan(-2x+\frac{\pi}{4})=\tan(2(\frac{\pi}{8}-x)) \Rightarrow T=\frac{4\pi}{8}$

$AC=\frac{\pi}{4}$ $\quad B=f(0)=\tan\frac{\pi}{4}=1$ $\quad S_{ABC}=\frac{1}{2}AC\times BO=\frac{\pi}{4}\times 1\times\frac{1}{2}=\frac{\pi}{4}$

(۲) ✓

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②

$x^2-(\sin\alpha)x-\frac{1}{4}\cos^2\alpha=0$, $f(\frac{2}{3})=0 \Rightarrow \frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}\cos^2\alpha=0$

$\Rightarrow 16-24\sin\alpha-9(1-\sin^2\alpha)=0 \Rightarrow 9\sin^2\alpha-24\sin\alpha+7=0 \Rightarrow \sin\alpha=\frac{24\pm\sqrt{324}}{18}$ $\begin{cases}\frac{42}{18} & \times\\ \frac{6}{18} & \checkmark\end{cases}$

$\Rightarrow \cos^2\alpha=1-\sin^2\alpha=\frac{8}{9} \Rightarrow 1+\tan^2\alpha=\frac{1}{\cos^2\alpha}=\frac{1}{\frac{8}{9}}=\frac{9}{8}$

(۲) ✓

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③

$\log(\sin\alpha)-\log(-\cos\alpha)=\log 3 \Rightarrow \log\frac{-\sin}{\cos}=\log 3 \Rightarrow -\tan\alpha=3$, (مثلث با اضلاع 1، 3، $\sqrt{10}$) $\tan=\frac{\text{مقابل}}{\text{مجاور}}=\frac{3}{1}$

$\Rightarrow \cos\alpha=\frac{-1}{\sqrt{10}}=\frac{-\sqrt{10}}{10}$, $\sin\alpha=\frac{3}{\sqrt{10}}=\frac{3\sqrt{10}}{10} \Rightarrow \sin\alpha-\cos\alpha=\frac{3\sqrt{10}}{10}+\frac{\sqrt{10}}{10}=\frac{4\sqrt{10}}{10}=\frac{2\sqrt{10}}{5}$

(۲) ✓

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④

$\sin^4\theta+\underbrace{\cos^2\theta}_{1-\sin^2}=\frac{4}{5}$ ① $\Rightarrow \sin^4\theta+1-\sin^2\theta=\frac{4}{5} \Rightarrow \sin^4\theta-\sin^2\theta=\frac{-1}{5}$

$\underbrace{(\sin^2\theta+\cos^2\theta)^2}_{1}=\sin^4+\underbrace{2\sin^2\theta\cos^2\theta}_{2\sin^2\theta-2\sin^4\theta}+\cos^4\theta=1 \Rightarrow \cos^4\theta+\sin^2\theta=1+\underbrace{\sin^4\theta-\sin^2\theta}_{\frac{-1}{5}}$

$\Rightarrow \cos^4\theta+\sin^2\theta=\frac{4}{5}$

(۲) ✓

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⑤

$\frac{3\sin\alpha+\sin\alpha\cos\alpha}{2-2\cos^2\alpha}<0$ و $\sin\alpha\tan\alpha-\frac{1}{\cos\alpha}>0$ ② $\sin\alpha\times\frac{\sin\alpha}{\cos\alpha}-\frac{1}{\cos\alpha}=\frac{\sin^2\alpha-1}{\cos\alpha}>0 \Rightarrow \cos\alpha<0$

صورت همواره منفی

① $\frac{\sin\alpha(3+\cos\alpha)}{2(\underbrace{1-\cos^2\alpha}_{\sin^2\alpha})}=\frac{3+\cos\alpha}{2\sin\alpha} \Rightarrow \sin\alpha<0$ (صورت همواره مثبت)

$\left.\begin{matrix}\sin\alpha<0\\ \cos\alpha<0\end{matrix}\right\}$ α ناحیه سوم

(۲) ✓

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⑥

$\frac{2}{\sin x}+\frac{3}{\cos x}=0 \Rightarrow \frac{2\cos x+3\sin x}{\sin x\cos x}=0 \xrightarrow{\text{صورت صفر}} 2\cos x=-3\sin x \Rightarrow -\frac{2\cos x}{3\sin x}=1$

$\Rightarrow \tan x=-\frac{2}{3} \Rightarrow \cot x=-\frac{3}{2} \Rightarrow \tan x+\cot x=-\frac{2}{3}+\frac{-3}{2}=\frac{-4-9}{6}=\frac{-15}{6}$ (۱۳)

✓ (۲)